

# Context-Free Parsing: The CKY Algorithm and Probabilistic Parsing

Introduction to Natural Language Processing  
Computer Science 585—Fall 2009  
University of Massachusetts Amherst

David Smith

# From Shift-Reduce to CKY

- Shift-reduce parsing can make wrong turns, needs backtracking
- Shift-reduce must pop the top of the stack, but how many items to pop?
- Time-space tradeoff
- Chomsky normal form

# Chomsky Normal Form

- Any CFL can be generated by an equivalent grammar in CNF
- Rules of three types
  - $X \rightarrow YZ$   $X, Y, Z$  nonterminals
  - $X \rightarrow a$   $X$  nonterminal,  $a$  terminal
  - $S \rightarrow \epsilon$   $S$  the start symbol
- NB: the derivation of a given string may change

# CNF Conversion

- Create new start symbol
- Remove NTs that can generate epsilon
- Remove NTs that can generate each other, (unary rule cycles)
- Chain rules with RHS  $> 2$
- Related topic: rule Markovization (later)

# CNF Conversion

Original

$$S \rightarrow TST \mid aB$$

$$T \rightarrow B \mid S$$

$$B \rightarrow b \mid \epsilon$$

# CNF Conversion

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$$S \rightarrow TST \mid aB$$

$$T \rightarrow B \mid S$$

$$B \rightarrow b \mid \epsilon$$

## New start symbol

$$S_0 \rightarrow S$$

$$S \rightarrow TST \mid aB$$

$$T \rightarrow B \mid S$$

$$B \rightarrow b \mid \epsilon$$

# CNF Conversion

Original

$$S \rightarrow TST \mid aB$$

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Remove nulls

$$S_0 \rightarrow S$$

$$S \rightarrow TST \mid aB \mid a \mid ST \mid TS$$

$$T \rightarrow B \mid S$$

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New start symbol

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## Remove nulls

$$S_0 \rightarrow S$$

$$S \rightarrow TST \mid aB \mid a \mid ST \mid TS$$

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## New start symbol

$$S_0 \rightarrow S$$

$$S \rightarrow TST \mid aB$$

$$T \rightarrow B \mid S$$

$$B \rightarrow b \mid \epsilon$$

## Remove unary cycles

$$S_0 \rightarrow TST \mid aB \mid a \mid ST \mid TS$$

$$S \rightarrow TST \mid aB \mid a \mid ST \mid TS$$

$$T \rightarrow b \mid TST \mid aB \mid a \mid ST \mid TS$$

$$B \rightarrow b$$



# CNF Conversion

$S_0 \rightarrow TST \mid UB \mid a \mid ST \mid TS$

$S \rightarrow TST \mid UB \mid a \mid ST \mid TS$

$T \rightarrow b \mid TST \mid UB \mid a \mid ST \mid TS$

$B \rightarrow b$

$U \rightarrow a$

**Make terminal  
rules unary**

# CNF Conversion

$$S_0 \rightarrow TST \mid UB \mid a \mid ST \mid TS$$

$$S \rightarrow TST \mid UB \mid a \mid ST \mid TS$$

$$T \rightarrow b \mid TST \mid UB \mid a \mid ST \mid TS$$

$$B \rightarrow b$$

$$U \rightarrow a$$

**Make terminal  
rules unary**

$$S_0 \rightarrow TZ \mid UB \mid a \mid ST \mid TS$$

$$S \rightarrow TZ \mid UB \mid a \mid ST \mid TS$$

$$T \rightarrow b \mid TZ \mid UB \mid a \mid ST \mid TS$$

$$B \rightarrow b$$

$$U \rightarrow a$$

$$Z \rightarrow ST$$

**Make non-terminal  
rules binary**

# CKY Algorithm

- Input: string of  $n$  words ( $m$  in M&S)
- Output (of recognizer): grammatical or not
- Dynamic programming in a **chart**:
  - rows labeled 0 to  $n-1$
  - columns labeled 1 to  $n$
  - cell  $[i,j]$  lists possible constituents spanning words between  $i$  and  $j$

# CKY Algorithm

- **for**  $i := 1$  to  $n$ 
  - Add to  $[i-1, i]$  all (part-of-speech) categories for the  $i^{\text{th}}$  word
- **for**  $\text{width} := 2$  to  $n$ 
  - **for**  $\text{start} := 0$  to  $n - \text{width}$ 
    - Define  $\text{end} := \text{start} + \text{width}$
    - **for**  $\text{mid} := \text{start} + 1$  to  $\text{end} - 1$ 
      - **for** every constituent  $X$  in  $[\text{start}, \text{mid}]$
      - **for** every constituent  $Y$  in  $[\text{mid}, \text{end}]$
      - **for** all ways of combining  $X$  and  $Y$  (if any)
      - Add the resulting constituent to  $[\text{start}, \text{end}]$  ~~if it's not already there.~~

# CKY Algorithm

- for  $i := 1$  to  $n$ 
  - Add to  $[i-1, i]$  all (part-of-speech) categories for the  $i^{\text{th}}$  word
- for width  $:= 2$  to  $n$ 
  - for start  $:= 0$  to  $n$ -width
    - Define end  $:=$  start + width
    - for mid  $:=$  start+1 to end-1
      - for every constituent  $X$  in  $[start, mid]$
      - for every constituent  $Y$  in  $[mid, end]$
      - for all ways of combining  $X$  and  $Y$  (if any)
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Time complexity?

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Time complexity?

$O(Gn^3)$

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Time complexity?

$O(Gn^3)$

Space complexity?

# CKY Algorithm

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      - for all ways of combining  $X$  and  $Y$  (if any)
      - Add the resulting constituent to  $[start, end]$  ~~if it's not already there.~~

Time complexity?

$O(Gn^3)$

Space complexity?

$O(Tn^2)$



time 1 flies 2 like 3 an 4 arrow 5

0	NP 3 Vst 3				
1		NP 4 VP 4			
2			P 2 V 5		
3				Det 1	
4					N 8

NP → time  
 Vst → time  
 NP → flies  
 VP → flies  
 P → like  
 V → like  
 Det → an  
 N → arrow

1 S → NP VP  
 6 S → Vst NP  
 2 S → S PP  
 1 VP → V NP  
 2 VP → VP PP  
 1 NP → Det N  
 2 NP → NP PP  
 3 NP → NP NP  
 0 PP → P NP

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0	NP 3 Vst 3				
1		NP 4 VP 4			
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3				Det 1	
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- 2 VP → VP PP
- 1 NP → Det N
- 2 NP → NP PP
- 3 NP → NP NP
- 0 PP → P NP

time 1 flies 2 like 3 an 4 arrow 5

0	NP 3 Vst 3	NP 10			
1		NP 4 VP 4			
2			P 2 V 5		
3				Det 1	
4					N 8

- 1 S → NP VP
- 6 S → Vst NP
- 2 S → S PP
- 1 VP → V NP
- 2 VP → VP PP
- 1 NP → Det N
- 2 NP → NP PP
- 3 NP → NP NP
- 0 PP → P NP

time 1 flies 2 like 3 an 4 arrow 5

0	NP 3 Vst 3	NP 10 S 8			
1		NP 4 VP 4			
2			P 2 V 5		
3				Det 1	
4					N 8

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- 2 NP → NP PP
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time 1 flies 2 like 3 an 4 arrow 5

0	NP 3 Vst 3	NP 10 S 8 S 13			
1		NP 4 VP 4			
2			P 2 V 5		
3				Det 1	
4					N 8

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time 1 flies 2 like 3 an 4 arrow 5

0	NP 3 Vst 3	NP 10 S 8 S 13			
1		NP 4 VP 4	-		
2			P 2 V 5	-	
3				Det 1	
4					N 8

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1		NP 4 VP 4	-		
2			P 2 V 5	-	
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1		NP 4 VP 4	-		
2			P 2 V 5	-	
3				Det 1	NP 10
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time 1 flies 2 like 3 an 4 arrow 5

0	NP 3 Vst 3	NP 10 S 8 S 13	-		
1		NP 4 VP 4	-	-	
2			P 2 V 5	-	PP 12
3				Det 1	NP 10
4					N 8

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time 1 flies 2 like 3 an 4 arrow 5

0	NP 3 Vst 3	NP 10 S 8 S 13	-		
1		NP 4 VP 4	-	-	
2			P 2 V 5	-	PP 12 VP 16
3				Det 1	NP 10
4					N 8

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1		NP 4 VP 4	-	-	
2			P 2 V 5	-	PP 12 VP 16
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time 1 flies 2 like 3 an 4 arrow 5

0	NP 3 Vst 3	NP 10 S 8 S 13	-	-	
1		NP 4 VP 4	-	-	NP 18
2			P 2 V 5	-	PP 12 VP 16
3				Det 1	NP 10
4					N 8

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0	NP 3 Vst 3	NP 10 S 8 S 13	-	-	
1		NP 4 VP 4	-	-	NP 18 S 21
2			P 2 V 5	-	PP 12 VP 16
3				Det 1	NP 10
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1		NP 4 VP 4	-	-	NP 18 S 21 VP 18
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time 1 flies 2 like 3 an 4 arrow 5

0	NP 3 Vst 3	NP 10 S 8 S 13	-	-	
1		NP 4 VP 4	-	-	NP 18 S 21 VP 18
2			P 2 V 5	-	PP 12 VP 16
3				Det 1	NP 10
4					N 8

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- 2 VP → VP PP
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time 1 flies 2 like 3 an 4 arrow 5

0	NP 3 Vst 3	NP 10 S 8 S 13	-	-	NP 24
1		NP 4 VP 4	-	-	NP 18 S 21 VP 18
2			P 2 V 5	-	PP 12 VP 16
3				Det 1	NP 10
4					N 8

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- 1 NP → Det N
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time 1 flies 2 like 3 an 4 arrow 5

0	NP 3 Vst 3	NP 10 S 8 S 13	-	-	NP 24 S 22
1		NP 4 VP 4	-	-	NP 18 S 21 VP 18
2			P 2 V 5	-	PP 12 VP 16
3				Det 1	NP 10
4					N 8

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time 1 flies 2 like 3 an 4 arrow 5

0	NP 3 Vst 3	NP 10 S 8 S 13	-	-	NP 24 S 22 S 27
1		NP 4 VP 4	-	-	NP 18 S 21 VP 18
2			P 2 V 5	-	PP 12 VP 16
3				Det 1	NP 10
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0	NP 3 Vst 3	NP 10 S 8 S 13	-	-	NP 24 S 22 S 27
1		NP 4 VP 4	-	-	NP 18 S 21 VP 18
2			P 2 V 5	-	PP 12 VP 16
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0	NP 3 Vst 3	NP 10 S 8 S 13	-	-	NP 24 S 22 S 27 NP 24
1		NP 4 VP 4	-	-	NP 18 S 21 VP 18
2			P 2 V 5	-	PP 12 VP 16
3				Det 1	NP 10
4					N 8

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0	NP 3 Vst 3	NP 10 S 8 S 13	-	-	NP 24 S 22 S 27 NP 24 S 27
1		NP 4 VP 4	-	-	NP 18 S 21 VP 18
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0	NP 3 Vst 3	NP 10 S 8 S 13	-	-	NP 24 S 22 S 27 NP 24 S 27 S 22
1		NP 4 VP 4	-	-	NP 18 S 21 VP 18
2			P 2 V 5	-	PP 12 VP 16
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1		NP 4 VP 4	-	-	NP 18 S 21 VP 18
2			P 2 V 5	-	PP 12 VP 16
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# Follow backpointers ... **S**

time 1 flies 2 like 3 an 4 arrow 5

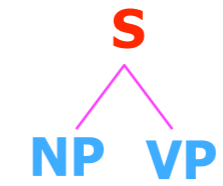
0	NP 3 Vst 3	NP 10 S 8 S 13	-	-	NP 24 <b>S 22</b> S 27 NP 24 S 27 S 22 S 27
1		NP 4 VP 4	-	-	NP 18 S 21 VP 18
2			P 2 V 5	-	PP 12 VP 16
3				Det 1	NP 10
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time 1 flies 2 like 3 an 4 arrow 5

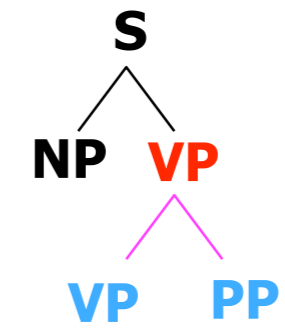
0	NP 3 Vst 3	NP 10 S 8 S 13	-	-	NP 24 S 22 S 27 NP 24 S 27 S 22 S 27
1		NP 4 VP 4	-	-	NP 18 S 21 VP 18
2			P 2 V 5	-	PP 12 VP 16
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time 1 flies 2 like 3 an 4 arrow 5

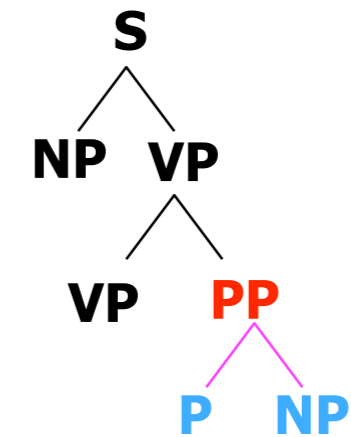
0	NP 3 Vst 3	NP 10 S 8 S 13	-	-	NP 24 S 22 S 27 NP 24 S 27 S 22 S 27
1		NP 4 VP 4	-	-	NP 18 S 21 VP 18
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time 1 flies 2 like 3 an 4 arrow 5

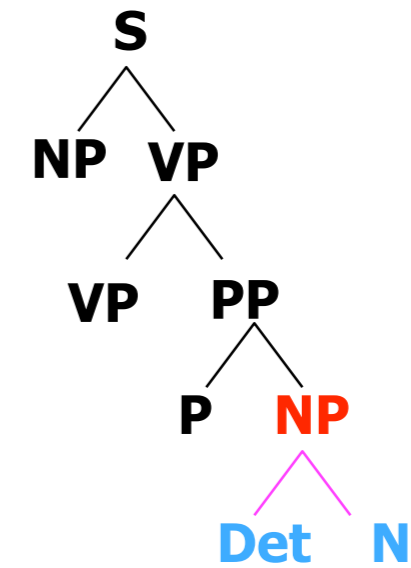
0	NP 3 Vst 3	NP 10 S 8 S 13	-	-	NP 24 S 22 S 27 NP 24 S 27 S 22 S 27
1		NP 4 VP 4	-	-	NP 18 S 21 VP 18
2			P 2 V 5	-	PP 12 VP 16
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time 1 flies 2 like 3 an 4 arrow 5

0	NP 3 Vst 3	NP 10 S 8 S 13	-	-	NP 24 S 22 S 27 NP 24 S 27 S 22 S 27
1		NP 4 VP 4	-	-	NP 18 S 21 VP 18
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# Parsing as Deduction

- CKY as inference rules
- CKY as Prolog program
- But Prolog is top-down with backtracking
  - i.e., “backward chaining”, CKY is “forward chaining”
- Inference rules as Boolean semiring

# Probabilistic CFGs

- Generative process (already familiar)
- It's context free: Rules are applied independently, therefore we multiply their probabilities
- How to estimate probabilities?
  - Supervised and unsupervised

# Questions for PCFGs

- What is the most likely parse for a sentence? (parsing)
- What is the probability of a sentence? (language modeling)
- What rule probabilities maximize the probability of a sentence? (parameter estimation)

# Algorithms for PCFGs

- Exact analogues to HMM algorithms
- Parsing: Viterbi CKY
- Language modeling: inside probabilities
- Parameter estimation: inside-outside probabilities with EM



# Parsing as Deduction

$\forall A, B, C \in N, W \in V, 0 \leq i, j, k \leq m$

$constit(B, i, j) \wedge constit(C, j, k) \wedge A \rightarrow BC \Rightarrow constit(A, i, k)$

$word(W, i) \wedge A \rightarrow W \Rightarrow constit(A, i, i + 1)$

## In Prolog:

```
constit(A, I1, I) :-  
  word(I, W),  
  (A ----> [W]),  
  I1 is I - 1.
```

```
constit(A, I, K) :-  
  constit(B, I, J),  
  constit(C, J, K),  
  (A ----> [B, C]).
```

*But Prolog uses top-down search with backtracking...*

# Parsing as Deduction

$$\forall A, B, C \in N, W \in V, 0 \leq i, j, k \leq m$$

$$\text{constit}(B, i, j) \wedge \text{constit}(C, j, k) \wedge A \rightarrow BC \Rightarrow \text{constit}(A, i, k)$$

$$\text{word}(W, i) \wedge A \rightarrow W \Rightarrow \text{constit}(A, i, i + 1)$$

$$\text{constit}(A, i, k) = \bigvee_{A, B, C, i, j, k} \text{constit}(B, i, j) \wedge \text{constit}(C, j, k) \wedge A \rightarrow BC$$

$$\text{constit}(A, i, i + 1) = \bigvee_{A, W, i} \text{word}(W, i) \wedge A \rightarrow W$$

# Parsing as Deduction

$$\mathit{constit}(A, i, k) = \bigvee_{A, B, C, i, j, k} \mathit{constit}(B, i, j) \wedge \mathit{constit}(C, j, k) \wedge A \rightarrow BC$$

$$\mathit{constit}(A, i, i + 1) = \bigvee_{A, W, i} \mathit{word}(W, i) \wedge A \rightarrow W$$

$$\begin{aligned} \mathit{score}(\mathit{constit}(A, i, k)) &= \max_{A, B, C, i, j, k} \mathit{score}(\mathit{constit}(B, i, j)) \\ &\quad \cdot \mathit{score}(\mathit{constit}(C, j, k)) \\ &\quad \cdot \mathit{score}(A \rightarrow BC) \end{aligned}$$

$$\mathit{score}(\mathit{constit}(A, i, i + 1)) = \max_{A, W, i} \mathit{score}(\mathit{word}(W, i)) \cdot \mathit{score}(A \rightarrow W)$$

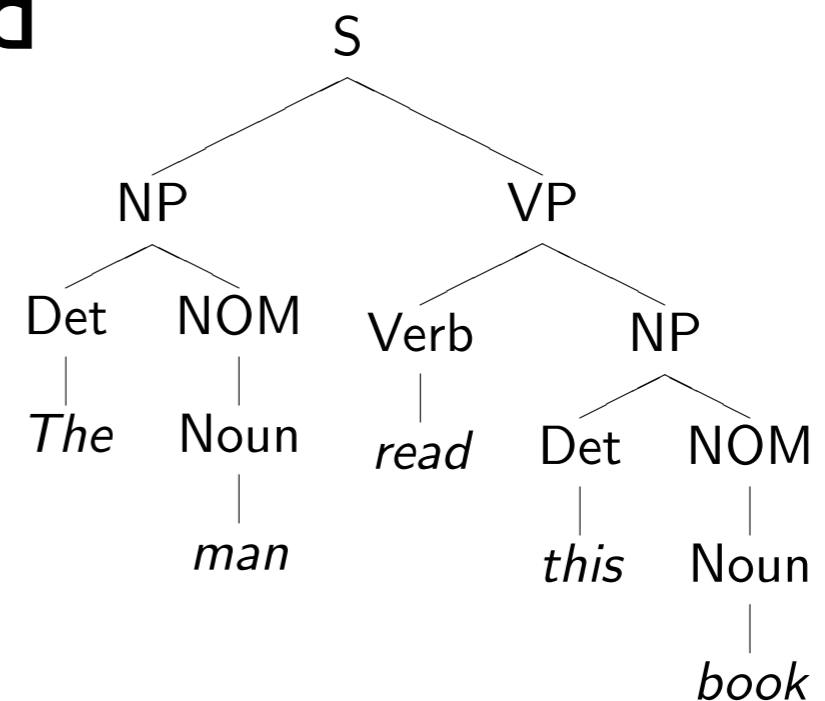
**And how about the inside algorithm?**

# Problems with Inside-Outside EM

- Each sentence at each iteration takes  $O(m^3n^3)$
- Local maxima even more problematic than for HMMs: Charniak (1993) found a different maximum for each of 300 trials
- More NTs needed to learn a good model
- NTs don't correspond to intuitions: HMMs are easier to constrain with tag dictionaries

# Treebank Grammars

- What rules would you extract from this tree?
- What probabilities would you assign them?



# Treebank Grammars

- Penn Treebank
- Lots of rules have high fanout (flat phrases)
- Lots of unary cycles
- How should we evaluate?
- What are the consequences of CNF conversion?