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CS4 HS

Higher/Lower Guessing Game: Winning via Efficient Searching

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Higher/Lower Guessing Game

Rules of Game:

- One player chooses a *number* between *minimum* and *maximum*.
- Other players try to guess the *number*. After each guess, they are told whether their guess is higher or lower than that *number*.
- The player who correctly guesses the *number* wins. If no one is correct after a specified number of guesses, the player who chose the *number* wins.

Teenage Mutant Ninja Turtles Fall of the Footclan™. Developed by Ultra Games, subsidiary of Konami. Copyright 1991. Rights belong to Ubisoft at present (2012).

Strategies:

- Use the higher/lower hints to change guessed number.
- Eliminate as many numbers as possible with a single guess and hint.

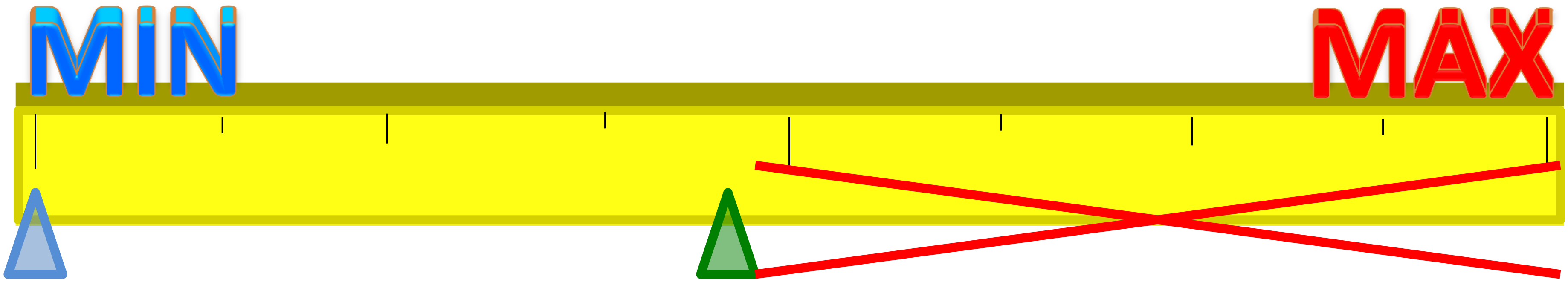
Searching for the Best Guess

Difficulty comes from number of guesses allowed.

- *(MAX – MIN* + 1*)* guesses is easiest. We can guess every number from *MIN* to *MAX* and always win. This is called a ***linear search***.
- What if we have fewer guesses? Can we still win every time? What are the fewest guesses that we need for this?

Consider:

- If guess is higher, then all numbers higher than guess are also higher. Same for lower.
- This eliminates some portion of the remaining guesses. What is the largest portion of guesses that we can remove regardless of higher/lower hint?
  - Answer: Half.

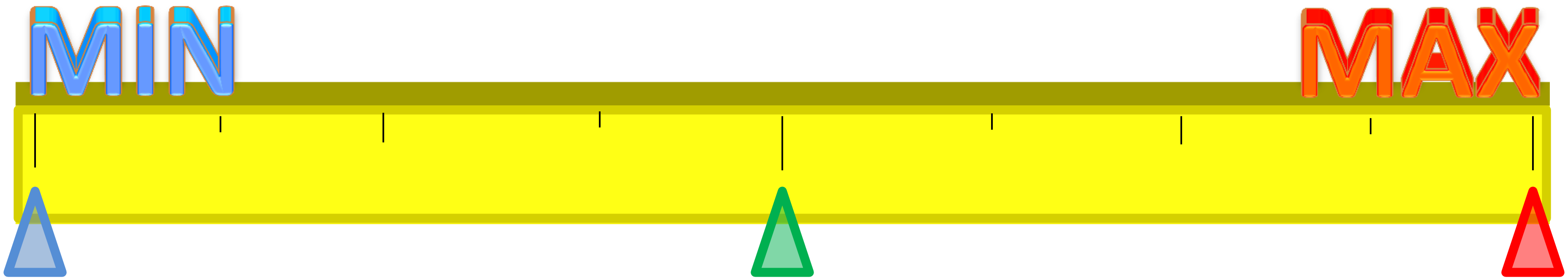


A Formula for Success

Perform ***binary search*** algorithm (by automation):

→ Begin with *MAXGUESS* = *MAX* and *MINGUESS* = *MIN*.

→ Guess the number halfway between *MAXGUESS* and *MINGUESS*.



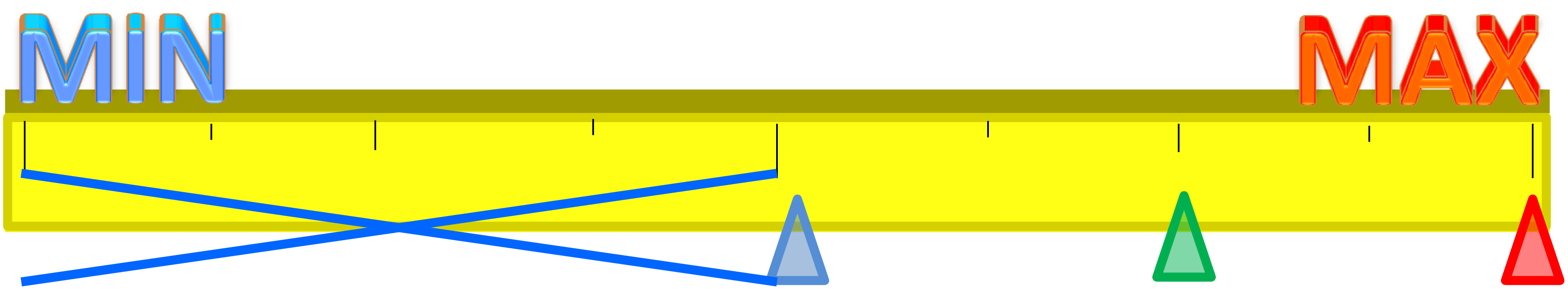
A Formula for Success

Perform ***binary search*** algorithm (by automation):

- Begin with *MAXGUESS* = *MAX* and *MINGUESS* = *MIN*.
- Guess the number halfway between *MAXGUESS* and *MINGUESS*.
  - If the guess is higher, then change *MAXGUESS* to 1 less than the guess itself.
  - If the guess is lower, then change *MINGUESS* to 1 more than the guess itself.
- Repeat the process by now guessing the new number halfway between *MAXGUESS* and *MINGUESS*.

Only half the numbers remain as possible guesses after each try.

- High school math can show that binary search will find the answer in no more than $\lceil \log_2(MAX - MIN + 1) \rceil$ guesses. This number is a lot smaller than ($MAX - MIN + 1$) as the interval gets bigger!



Rematch: Let's Try It Out!

$\lceil \log_2(MAX - MIN + 1) \rceil = \lceil \log_2(999 - 0 + 1) \rceil = \lceil \log_2(1000) \rceil = 10$ … How convenient!

MINGUESS	*MAXGUESS*	*HALFWAY*
0	999	500
0	499	250
251	499	375
251	374	312
251	311	281
251	280	265
251	264	257
251	256	253
254	256	255
254	254	254

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Further Computational Thinking

If a group of objects can be ordered, we can use binary search instead of linear search.

- What is the best way to order the objects? Are there some ways that are better for certain situations?
  - Numbers: least to greatest
  - Words: lexicographically
- How do we know if binary search fails (something is not in the group)?

Is there a quicker way to search than binary search?

- What conditions must be satisfied for such a method to work?

How do we put the objects in order to perform binary search?

- What are we sorting?
- Are there duplicate objects in the group?
- What are our memory and/or time limits?