

The Complexity of Resilience and Responsibility for Conjunctive Queries

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to appear in VLDB 2016
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Treat exogenous part as fixed, beyond our control; only consider possible changes to the endogenous part.

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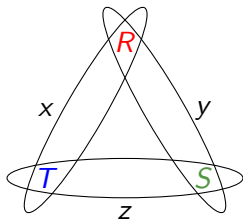
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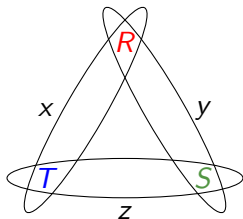
Goal: Characterize the **complexity** of **resilience** for **sj-free conjunctive queries**.

Triangle Query



$$q_{\Delta} :- R(x, y), S(y, z), T(z, x)$$

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Query **hypergraph**: relations are vertices;
variables are hyperedges

The triangle query, q_{Δ} , is hard.

Prop. $\text{RES}(q_{\Delta})$ is NP-complete.

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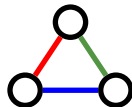
Proof: Reduce 3SAT to $\text{RES}(q_\Delta)$. Let $\psi = C_1 \wedge \dots \wedge C_m$ be a 3-CNF formula, $\text{var}(\psi) = \{v_1, \dots, v_n\}$

Map $\psi \mapsto (D_\psi, k_\psi)$ s.t. $\psi \in 3\text{SAT} \Leftrightarrow (D_\psi, k_\psi) \in \text{RES}(q_\Delta)$

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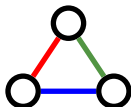
$$(D_\psi, k_\psi) \in \text{RES}(q_\Delta) \Leftrightarrow \exists \Gamma \mid \Gamma \mid = k_\psi \wedge D_\psi - \Gamma \text{ has no}$$



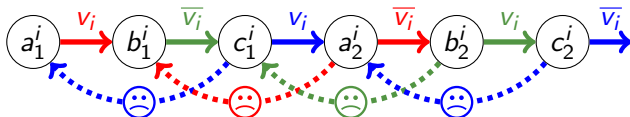
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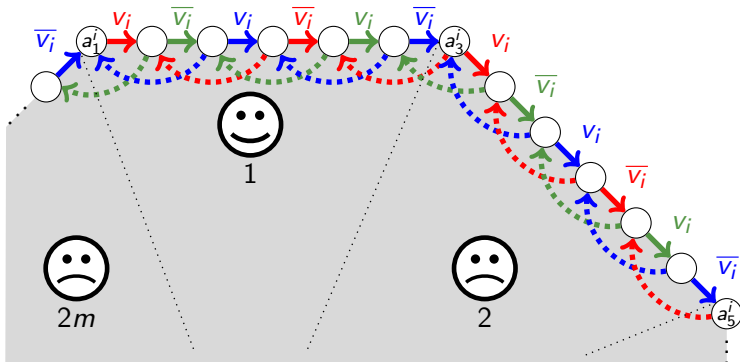
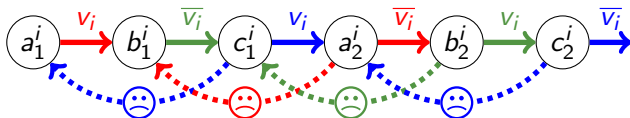
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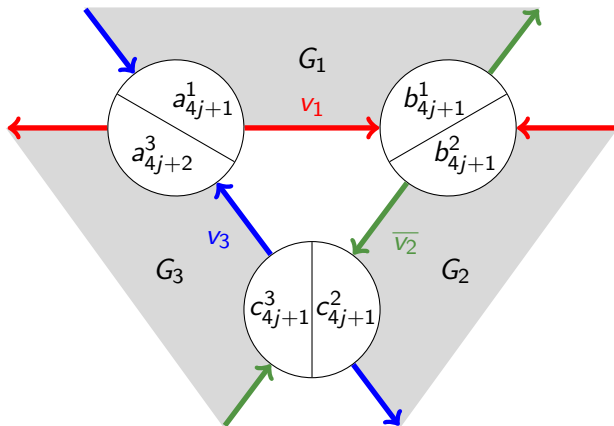
D_ψ has one circular gadget G_i for each variable v_i .



In G_i must choose all v_i 's or all \bar{v}_i 's



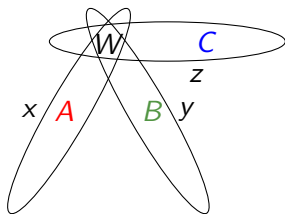
For each clause, e.g., $C_j = (v_1 \vee \bar{v}_2 \vee v_3)$, pick the j th occurrences of $v_1 \in G_1$, $\bar{v}_2 \in G_2$ and $v_3 \in G_3$. Identify head of v_1 with tail of \bar{v}_2 , head of \bar{v}_2 with tail of v_3 , head of v_3 with tail of v_1



This new RGB triangle is automatically removed iff one of the literals in C_j is chosen true.

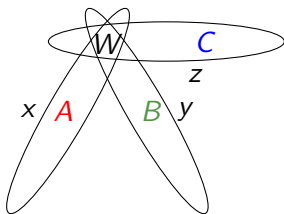


Tripod Query



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$$A = \{ \langle ab \rangle \mid R(a, b) \in D \}$$

$$B = \{ \langle bc \rangle \mid S(b, c) \in D \}$$

$$C = \{ \langle ca \rangle \mid T(c, a) \in D \}$$

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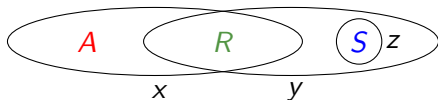
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Claim $(D, k) \in \text{RES}(q_\Delta) \iff (D', k) \in \text{RES}(q_T)$. □

Def. A query is **linear** if all of the vertices of its hypergraph can be drawn along a straight line with all of its hyperedges convex.

For example, the following query is linear:

$$q :- A(x), R(x, y), S(y, z)$$



Linear Queries are Easy

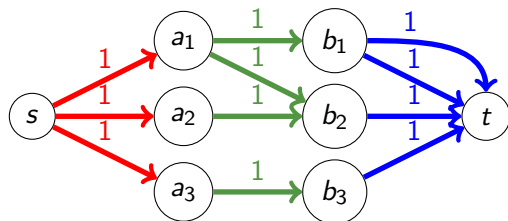
Prop. For any linear sj-free conjunctive query q , $\text{RES}(q) \in \text{P}$.

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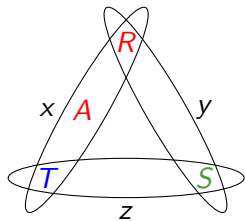
Proof: Use Network Flow.

$\text{RES}(D, q)$ is the min cut of corresponding network.

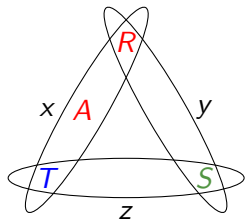


$q:-$ $A(x)$ $R(x, y)$ $S(y, z)$



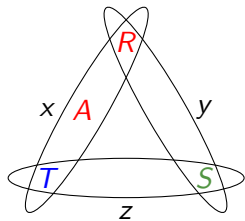


$$q_{\text{rats}} \quad :- \quad A(x), R(x, y), S(y, z), T(z, x)$$



Is Rats **hard** or **easy** ?

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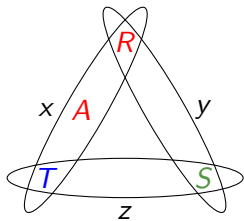
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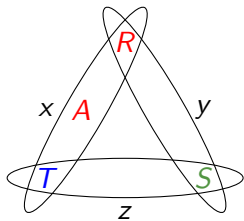
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Domination



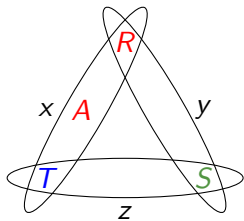
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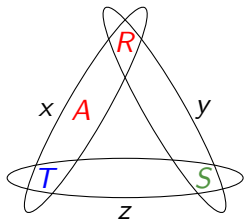
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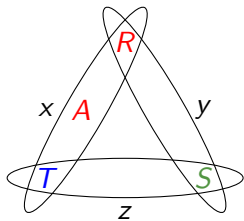
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q_4	\equiv	$A(x), R^x(x, y, z), S(y, z)$	Repetition
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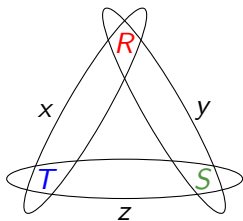
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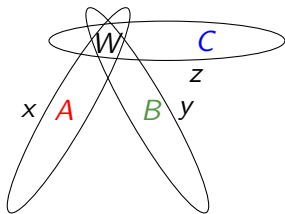
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What do the triangle and the tripod have in common?

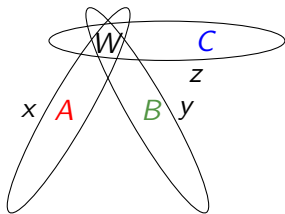
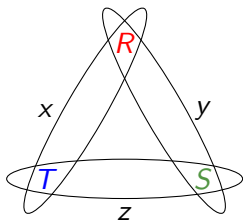


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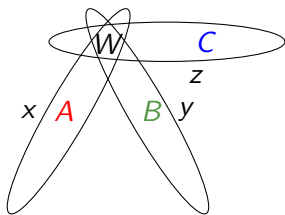
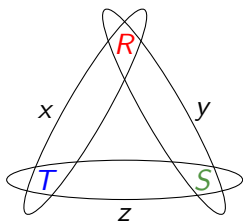
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$$q_{\Delta} := R(x, y), S(y, z), T(z, x) \quad q_T := A(x), B(y), C(z), W^x(x, y, z)$$

Def. A **triad** is a set of three endogenous atoms, $\mathcal{T} = \{S_0, S_1, S_2\}$ such that for every pair i, j , there is a path from S_i to S_j that uses no variable occurring in the other atom of \mathcal{T} .

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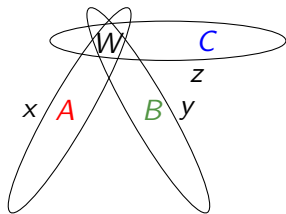
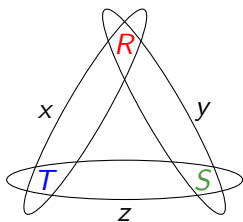


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Proof: Show $\text{RES}(q_{\Delta}) \leq \text{RES}(q)$



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Proof: By induction on the number of *endogenous* atoms in q
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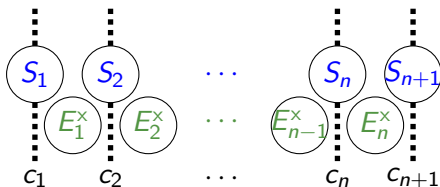
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Since there is no triad, we can linearize the endogenous atoms:



Dichotomy Theorem for Resilience: Let q be a sj-free conjunctive query all of whose dominated atoms are exogenous. If q has a triad then $\text{RES}(q)$ is NP complete. Otherwise, $\text{RES}(q) \in \text{P}$.

Extend to Databases with Functional Dependencies

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Corollary Induced rewrites characterized the effect of FD's:

$$\text{RES}(q; \Phi) \equiv \text{RES}(q^*; \Phi) \equiv \text{RES}(q^*)$$

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- ▶ **Understand & explain Dichotomy Phenomenon**