

# Lecture 2: Context Free Grammars

Introduction to Natural Language Processing

CS 585

Fall 2004

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Also includes material from Chris Manning.

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# Today's Main Points

- Demo of “hands on” with text, using Unix tools  
Zipf's Law
- A brief introduction to syntax in NLP.
- Define context free grammars.  
Give some examples.
- Chomsky normal form. Converting to it.
- Parsing as search  
Top-down, bottom up, and the problems with each.
- Hand out homework #1.
- Read M&S Ch 3, if you haven't already.

# Administration

- I will be giving research presentations in D.C. next Tuesday & Thursday.
- Tuesday: Wei Li with review of probability
- Thursday: Aron Culotta: information theory and naive Bayes.
- Be sure to subscribe to cs585 mailing list, if you haven't already.  
I sent a test message Sunday night.

## “Hands on” with Text

- Get a large body of electronic text (called a *corpus*).
- Browse it.
- Use `perl` and other Unix tools to look at counts of words.  
What does this distribution look like?

Messy, but the following works:

```
cat * | perl -pe 's/[^a-zA-Z]/\n/g' | egrep '.' \
| awk '{print tolower($0)}' \
| sort | uniq -c | sort -nr | less
```

```
cat sawyr10.txt | perl -pe 's/[^a-zA-Z]/\n/g' | egrep '.' \
| awk '{print tolower($0)}' \
| sort | uniq -c | sort -nr | \
perl -pe 's/[^0-9\n]//g' | uniq -c | less
```

## Word frequencies in Tom Sawyer

| Word | Freq | Use                                   |
|------|------|---------------------------------------|
| the  | 3332 | determiner (article)                  |
| and  | 2972 | conjunction                           |
| a    | 1775 | determiner                            |
| to   | 1725 | preposition, verbal infinitive marker |
| of   | 1440 | preposition                           |
| was  | 1161 | auxiliary verb                        |
| it   | 1027 | (personal/expletive) pronoun          |
| in   | 906  | preposition                           |
| that | 877  | complementizer, demonstrative         |
| he   | 877  | (personal) pronoun                    |
| I    | 783  | (personal) pronoun                    |
| his  | 772  | (possessive) pronoun                  |
| you  | 686  | (personal) pronoun                    |
| Tom  | 679  | proper noun                           |
| with | 642  | preposition                           |

## Frequencies of frequencies in Tom Sawyer

| Word Frequency | Frequency of Frequency |
|----------------|------------------------|
| 1              | 3993                   |
| 2              | 1292                   |
| 3              | 664                    |
| 4              | 410                    |
| 5              | 243                    |
| 6              | 199                    |
| 7              | 172                    |
| 8              | 131                    |
| 9              | 82                     |
| 10             | 91                     |
| 11-50          | 540                    |
| 51-100         | 99                     |
| >100           | 102                    |

71,730 word tokens

8,018 word types

## Zipf's Law in Tom Sawyer

| Word  | Freq. (f) | Rank (r) | f * r |
|-------|-----------|----------|-------|
| the   | 3332      | 1        | 3332  |
| and   | 2972      | 2        | 5944  |
| a     | 1775      | 3        | 5235  |
| he    | 877       | 10       | 8770  |
| but   | 710       | 20       | 8400  |
| be    | 294       | 30       | 8820  |
| there | 222       | 40       | 8880  |
| one   | 172       | 50       | 8600  |
| about | 158       | 60       | 9480  |
| more  | 138       | 70       | 9480  |
| never | 124       | 80       | 9920  |
| Oh    | 116       | 90       | 10440 |
| two   | 104       | 100      | 10400 |

## Zipf's Law in Tom Sawyer

| Word       | Freq. (f) | Rank (r) | f * r |
|------------|-----------|----------|-------|
| turned     | 51        | 200      | 10200 |
| youll      | 30        | 300      | 9000  |
| name       | 21        | 400      | 8400  |
| comes      | 16        | 500      | 8000  |
| group      | 13        | 600      | 7800  |
| lead       | 11        | 700      | 7700  |
| friends    | 10        | 800      | 8000  |
| begin      | 9         | 900      | 8100  |
| family     | 8         | 1000     | 8000  |
| brushed    | 4         | 2000     | 8000  |
| sins       | 2         | 3000     | 6000  |
| Could      | 2         | 4000     | 8000  |
| Applausive | 1         | 8000     | 8000  |



# Zipf's Law

$$\text{frequency} \propto \frac{1}{\text{rank}}$$

In other words, there is a constant,  $k$ , such that  $f \cdot r = k$ .

# Language structure and meaning

We want to know how meaning is mapped onto what language structures. Commonly in English in ways like this:

[THING The dog] is [PLACE in the garden]

[THING The dog] is [PROPERTY fierce]

[ACTION [THING The dog] is chasing [THING the cat]]

[STATE [THING The dog] was sitting [PLACE in the garden] [TIME yesterday]]

[ACTION [THING We] ran [PATH out into the water]]

[ACTION [THING The dog] barked [PROPERTY/MANNER loudly]]

[ACTION [THING The dog] barked [PROPERTY/AMOUNT nonstop for five hours]]

## Word categories: Traditional parts of speech

|              |                      |                      |
|--------------|----------------------|----------------------|
| Noun         | Names of things      | boy, cat, truth      |
| Verb         | Action or state      | become, hit          |
| Pronoun      | Used for noun        | I, you, we           |
| Adverb       | Modifies V, Adj, Adv | sadly, very          |
| Adjective    | Modifies noun        | happy, clever        |
| Conjunction  | Joins things         | and, but, while      |
| Preposition  | Relation of N        | to, from, into       |
| Interjection | An outcry            | ouch, oh, alas, psst |

## Part of speech “Substitution Test”

The {sad, intelligent, green, fat, ...} one is in the corner.

# Constituency

The idea: Groups of words may behave as a single unit or phrase, called a **constituent**.

E.g. Noun Phrase

*Kermit the frog*

*they*

*December twenty-sixth*

*the reason he is running for president*

# Constituency

Sentences have parts, some of which appear to have subparts. These groupings of words that go together we will call constituents.

(How do we know they go together? Coming in a few slides...)

*I hit the man with a cleaver*

I hit [the man with a cleaver]

I hit [the man] with a cleaver

*You could not go to her party*

You [could not] go to her party

You could [not go] to her party

## Constituent Phrases

For constituents, we usually name them as phrases based on the word that heads the constituent:

|                             |  |
|-----------------------------|--|
| <i>the man from Amherst</i> | is a Noun Phrase (NP) because the head <i>man</i> is a noun                  |
| <i>extremely clever</i>     | is an Adjective Phrase (AP) because the head <i>clever</i> is an adjective   |
| <i>down the river</i>       | is a Prepositional Phrase (PP) because the head <i>down</i> is a preposition |
| <i>killed the rabbit</i>    | is a Verb Phrase (VP) because the head <i>killed</i> is a verb               |

Note that a word is a constituent (a little one). Sometimes words also act as phrases. In:

*Joe grew potatoes.*

*Joe* and *potatoes* are both nouns and noun phrases.

Compare with:

*The man from Amherst* *grew* *beautiful russet potatoes*.

We say *Joe* counts as a noun phrase because it appears in a place that a larger noun phrase could have been.

## Evidence constituency exists

1. They appear in similar environments (before a verb)

Kermit the frog comes on stage

They come to Massachusetts every summer

December twenty-sixth comes after Christmas

The reason he is running for president comes out only now.

But not each individual word in the constituent

\*The comes out... \*is comes out... \*for comes out...

2. The constituent can be placed in a number of different locations

Constituent = Prepositional phrase: On December twenty-sixth

On December twenty-sixth I'd like to fly to Florida.

I'd like to fly on December twenty-sixth to Florida.

I'd like to fly to Florida on December twenty-sixth.

But not split apart

\*On December I'd like to fly twenty-sixth to Florida.

\*On I'd like to fly December twenty-sixth to Florida.



# Context-free grammar

The most common way of modeling constituency.

CFG = Context-Free Grammar = Phrase Structure Grammar  
= BNF = Backus-Naur Form

The idea of basing a grammar on constituent structure dates back to Wilhem Wundt (1890), but not formalized until Chomsky (1956), and, independently, by Backus (1959).

# Context-free grammar

$$G = \langle T, N, S, R \rangle$$

- $T$  is set of terminals (lexicon)
- $N$  is set of non-terminals For NLP, we usually distinguish out a set  $P \subset N$  of *preterminals* which always rewrite as terminals.
- $S$  is start symbol (one of the nonterminals)
- $R$  is rules/productions of the form  $X \rightarrow \gamma$ , where  $X$  is a nonterminal and  $\gamma$  is a sequence of terminals and nonterminals (may be empty).
- A grammar  $G$  generates a language  $L$ .

## An example context-free grammar

$G = \langle T, N, S, R \rangle$

$T = \{that, this, a, the, man, book, flight, meal, include, read, does\}$

$N = \{S, NP, NOM, VP, Det, Noun, Verb, Aux\}$

$S = S$

$R = \{$

$S \rightarrow NP VP$

$S \rightarrow Aux NP VP$

$S \rightarrow VP$

$NP \rightarrow Det NOM$

$NOM \rightarrow Noun$

$NOM \rightarrow Noun NOM$

$VP \rightarrow Verb$

$VP \rightarrow Verb NP$

$Det \rightarrow that \mid this \mid a \mid the$

$Noun \rightarrow book \mid flight \mid meal \mid man$

$Verb \rightarrow book \mid include \mid read$

$Aux \rightarrow does$

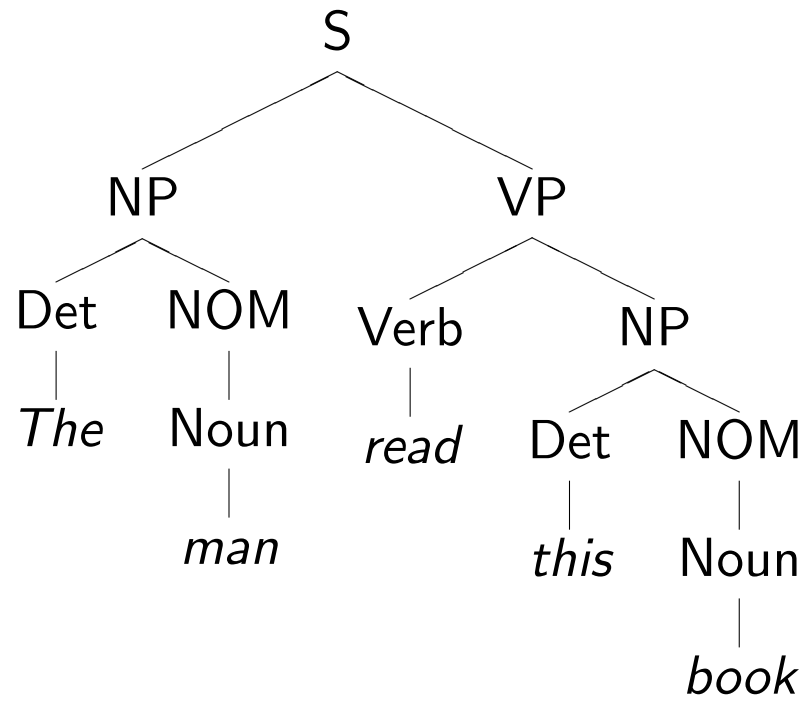
$\}$

## Application of grammar rewrite rules

|                            |  |
|----------------------------|--|
| $S \rightarrow NP VP$      | $Det \rightarrow that \mid this \mid a \mid the$       |
| $S \rightarrow Aux NP VP$  | $Noun \rightarrow book \mid flight \mid meal \mid man$ |
| $S \rightarrow VP$         | $Verb \rightarrow book \mid include \mid read$         |
| $NP \rightarrow Det NOM$   | $Aux \rightarrow does$                                 |
| $NOM \rightarrow Noun$     |  |
| $NOM \rightarrow Noun NOM$ |  |
| $VP \rightarrow Verb$      |  |
| $VP \rightarrow Verb NP$   |  |

$S \rightarrow NP VP$   
→ Det NOM VP  
→ *The* NOM VP  
→ *The* Noun VP  
→ *The man* VP  
→ *The man* Verb NP  
→ *The man read* NP  
→ *The man read* Det NOM  
→ *The man read this* NOM  
→ *The man read this* Noun  
→ *The man read this book*

# Parse tree



## CFGs can capture recursion

Example of seemingly endless recursion of embedded prepositional phrases:

PP → Prep NP

NP → Noun PP

[*S* The shepherds ate their [*NP* lunch [*PP* of potatoes [*PP* from the fields [*PP* of the farm [*PP* by the bend [*PP* in the river]]]]]]].

(Bracket notation)

# Grammaticality

A CFG defines a formal language = the set of all sentences (strings of words) that can be derived by the grammar.

Sentences in this set said to be **grammatical**.

Sentences outside this set said to be **ungrammatical**.

# The Chomsky hierarchy

- Type 0 Languages / Grammars

Rewrite rules  $\alpha \rightarrow \beta$

where  $\alpha$  and  $\beta$  are any string of terminals and nonterminals

- Context-sensitive Languages / Grammars

Rewrite rules  $\alpha X \beta \rightarrow \alpha \gamma \beta$

where  $X$  is a non-terminal, and  $\alpha, \beta, \gamma$  are any string of terminals and nonterminals, ( $\gamma$  must be non-empty).

- Context-free Languages / Grammars

Rewrite rules  $X \rightarrow \gamma$

where  $X$  is a nonterminal and  $\gamma$  is any string of terminals and nonterminals

- Regular Languages / Grammars

Rewrite rules  $X \rightarrow \alpha Y$

where  $X, Y$  are single nonterminals, and  $\alpha$  is a string of terminals;  $Y$  might be missing.



## Parsing regular grammars

(Languages that can be generated by finite-state automata.)

Finite state automaton  $\leftrightarrow$  regular expression  $\leftrightarrow$  regular grammar

Space needed to parse: constant

Time needed to parse: linear (in the length of the input string)

Cannot do embedded recursion, e.g.  $a^n b^n$ . (Context-free grammars can.)  
ab, aaabbb, \*aabbb

The cat likes tuna fish.

The cat the dog chased likes tuna fish

The cat the dog the boy loves chased likes tuna fish.

John, always early to rise, even after a sleepless night filled with the cries of the neighbor's baby, goes running every morning.

John and Mary, always early to rise, even after a sleepless night filled with the cries of the neighbor's baby, go running every morning.

# Parsing context-free grammars

(Languages that can be generated by pushdown automata.)

Widely used for surface syntax description (correct word order specification) in natural languages.

Space needed to parse: stack (sometimes a stack of stacks)

In general, proportional to the number of levels of recursion in the data.

Time needed to parse: in general  $O(n^3)$ .

Can do  $a^n b^n$ , but cannot do  $a^n b^n c^n$ .

## Chomsky Normal Form

All rules of the form  $X \rightarrow YZ$  or  $X \rightarrow a$ .

Any CFG can be converted into this form.

How would you convert the rule  $W \rightarrow XYaZ$  to Chomsky Normal Form?

## Parsing context-sensitive grammars

(Languages that can be recognized by a non-deterministic Turing machine whose tape is bounded by a constant times the length of the input.)

Natural languages are really not context-free: e.g. pronouns more likely in Object rather than Subject of a sentence.

But parsing is PSPACE-complete! (Recognized by a Turing machine using a polynomial amount of memory, and unlimited time.)

Often work with *mildly* context-sensitive grammars. More on this next week. E.g. Tree-adjoining grammars. Time needed to parse, e.g.  $O(n^6)$  or  $O(n^5)$ ...

## Bottom-up versus Top-down science

- **empiricist**

Britain: Francis Bacon, John Locke

Knowledge is induced and reasoning proceeds based on data from the real world.

- **rationalist**

Continental Europe: Descartes

Learning and reasoning is guided by prior knowledge and innate ideas.

# What is parsing?

We want to run the grammar backwards to find the structure.

Parsing can be viewed as a search problem.

We search through the legal rewritings of the grammar.

We want to find *all* structures matching an input string of words (for the moment)

We can do this bottom-up or top-down

This distinction is independent of depth-first versus breadth-first; we can do either both ways.

Doing this we build a *search tree* which is different from the *parse tree*.

# Recognizers and parsers

- A **recognizer** is a program for which a given grammar and a given sentence returns YES if the sentence is accepted by the grammar (i.e., the sentence is in the language), and NO otherwise.
- A **parser** in addition to doing the work of a recognizer also returns the set of parse trees for the string.

## Soundness and completeness

- A parser is **sound** if every parse it returns is valid/correct.
- A parser **terminates** if it is guaranteed not to go off into an infinite loop.
- A parser is **complete** if for any given grammar and sentence it is sound, produces every valid parse for that sentence, and terminates.
- (For many cases, we settle for sound but incomplete parsers: e.g. probabilistic parsers that return a  $k$ -best list.)

# Top-down parsing

Top-down parsing is goal-directed.

- A top-down parser starts with a list of constituents to be built.
- It rewrites the goals in the goal list by matching one against the LHS of the grammar rules,
- and expanding it with the RHS,
- ...attempting to match the sentence to be derived.

If a goal can be rewritten in several ways, then there is a choice of which rule to apply (search problem)

Can use depth-first or breadth-first search, and goal ordering.



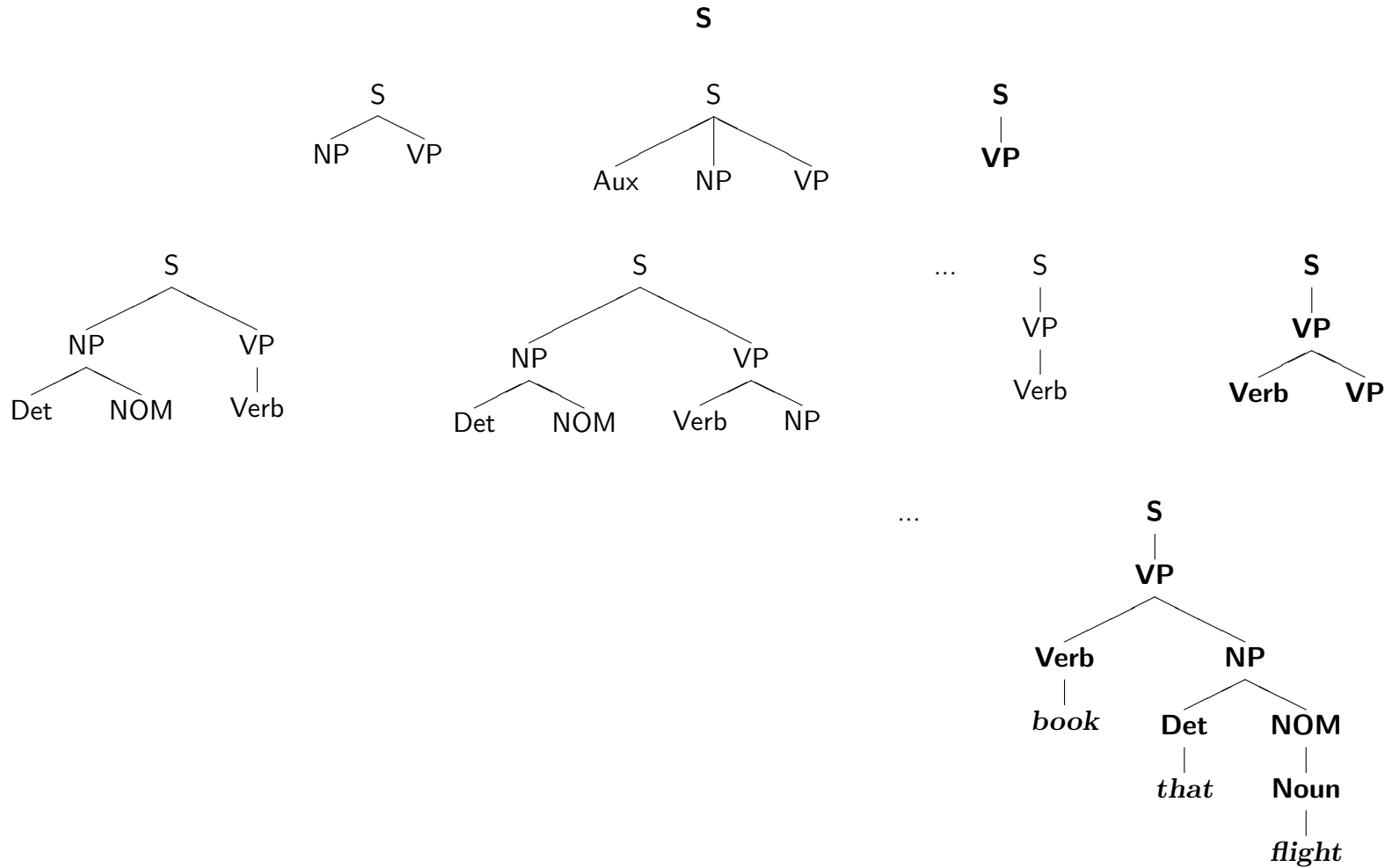
## Top-down parsing example (Breadth-first)

|                |   |
|----------------|---|
| S → NP VP      | Det → <i>that</i>   <i>this</i>   <i>a</i>   <i>the</i>       |
| S → Aux NP VP  | Noun → <i>book</i>   <i>flight</i>   <i>meal</i>   <i>man</i> |
| S → VP         | Verb → <i>book</i>   <i>include</i>   <i>read</i>             |
| NP → Det NOM   | Aux → <i>does</i>   |
| NOM → Noun     |   |
| NOM → Noun NOM |   |
| VP → Verb      |   |
| VP → Verb NP   |   |

*Book that flight.*

(Work out top-down, breadth-first search on the board...)

# Top-down parsing example (Breadth-first)



## Problems with top-down parsing

- Left recursive rules... e.g.  $NP \rightarrow NP PP$ ... lead to infinite recursion
- Will do badly if there are many different rules for the same LHS. Consider if there are 600 rules for S, 599 of which start with NP, but one of which starts with a V, and the sentence starts with a V.
- Useless work: expands things that are possible top-down but not there (no bottom-up evidence for them).
- Top-down parsers do well if there is useful grammar-driven control: search is directed by the grammar.
- Top-down is hopeless for rewriting parts of speech (preterminals) with words (terminals). In practice that is always done bottom-up as lexical lookup.
- Repeated work: anywhere there is common substructure.

# Bottom-up parsing

Top-down parsing is data-directed.

- The initial goal list of a bottom-up parser is the string to be parsed.
- If a sequence in the goal list matches the RHS of a rule, then this sequence may be replaced by the LHS of the rule.
- Parsing is finished when the goal list contains just the start symbol.

If the RHS of several rules match the goal list, then there is a choice of which rule to apply (search problem)

Can use depth-first or breadth-first search, and goal ordering.

The standard presentation is as **shift-reduce** parsing.

## Bottom-up parsing example

|                            |  |
|----------------------------|--|
| $S \rightarrow NP VP$      | $Det \rightarrow that \mid this \mid a \mid the$       |
| $S \rightarrow Aux NP VP$  | $Noun \rightarrow book \mid flight \mid meal \mid man$ |
| $S \rightarrow VP$         | $Verb \rightarrow book \mid include \mid read$         |
| $NP \rightarrow Det NOM$   | $Aux \rightarrow does$                                 |
| $NOM \rightarrow Noun$     |  |
| $NOM \rightarrow Noun NOM$ |  |
| $VP \rightarrow Verb$      |  |
| $VP \rightarrow Verb NP$   |  |

*Book that flight.*

(Work out bottom-up search on the board...)

## Shift-reduce parsing

| Stack             | Input remaining  | Action  |
|-------------------|------------------|---|
| ( )               | Book that flight | shift   |
| (Book)            | that flight      | reduce, Verb $\rightarrow$ book, (Choice #1 of 2) |
| (Verb)            | that flight      | shift   |
| (Verb that)       | flight           | reduce, Det $\rightarrow$ that                    |
| (Verb Det)        | flight           | shift   |
| (Verb Det flight) |                  | reduce, Noun $\rightarrow$ flight                 |
| (Verb Det Noun)   |                  | reduce, NOM $\rightarrow$ Noun                    |
| (Verb Det NOM)    |                  | reduce, NP $\rightarrow$ Det NOM                  |
| (Verb NP)         |                  | reduce, VP $\rightarrow$ Verb NP                  |
| (Verb)            |                  | reduce, S $\rightarrow$ V                         |
| (S)               |                  | SUCCESS!  |

Ambiguity may lead to the need for backtracking.

# Shift Reduce Parser

Start with the sentence to be parsed in an input buffer.

- a "shift" action corresponds to pushing the next input symbol from the buffer onto the stack
- a "reduce" action occurs when we have a rule's RHS on top of the stack. To perform the reduction, we pop the rule's RHS off the stack and replace it with the terminal on the LHS of the corresponding rule.

If you end up with only the `START` symbol on the stack, then success!

If you don't, and you cannot and no "shift" or "reduce" actions are possible, backtrack.

# Shift Reduce Parser

In a top-down parser, the main decision was which production rule to pick.  
In a bottom-up shift-reduce parser there are two decisions:

1. Should we shift another symbol, or reduce by some rule?
2. If reduce, then reduce by which rule?

both of which can lead to the need to backtrack



## Problems with bottom-up parsing

- Unable to deal with empty categories: termination problem, unless rewriting empties as constituents is somehow restricted (but then it's generally incomplete)
- Useless work: locally possible, but globally impossible.
- Inefficient when there is great lexical ambiguity (grammar-driven control might help here). Conversely, it is data-directed: it attempts to parse the words that are there.
- Repeated work: anywhere there is common substructure.
- Both Top-down (LL) and Bottom-up (LR) parsers can (and frequently do) do work exponential in the sentence length on NLP problems.

## Principles for success

- Left recursive structures must be found, not predicted.
- Empty categories must be predicted, not found.
- Don't waste effort re-working what was previously parsed before backtracking.

### **An alternative way to fix things:**

- Grammar transformations can fix both left-recursion and epsilon productions.
- Then you parse the same language but with different trees.
- BUT linguists tend to hate you, because the structure of the re-written grammar isn't what they wanted.

## Coming next...

A dynamic programming solution for parsing: CYK and Earley's Algorithm.

(Then later in the semester.)

Probabilistic version of these models. Find the most *likely* parse when several are possible.