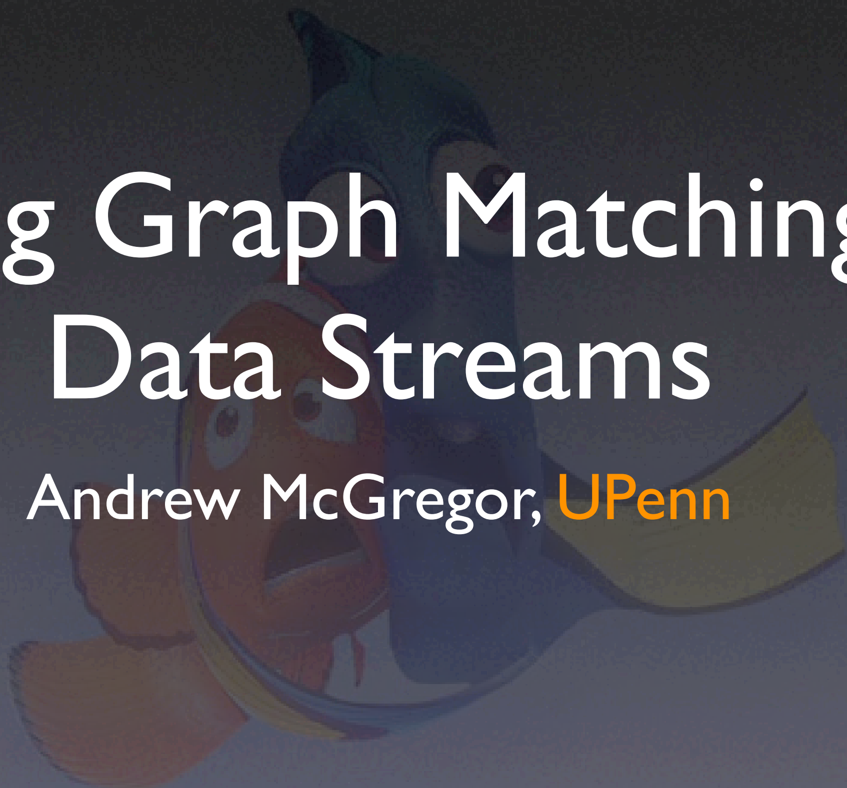

Finding Graph Matchings in Data Streams

Andrew McGregor, UPenn

The Streaming Model

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- Classic Problem: Median Finding [Munro & Paterson]

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  - How many passes?
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- Statistics, Norms and Histograms…
- What about graph problems?

Graph Streaming

- Instance of graph problem $G = (V, E)$
- Edges arrive in arbitrary order: $e_1, e_2, e_3, \ldots, e_m$
- Memory limit $O(n \text{ polylog } n)$ where $n = |V|$

- Spanner Construction, Bipartite Matching, Lower Bounds [Feigenbaum, Kannan, M. , Suri, Zhang '04 &'05]
- "Annotation" Stream Model [Aggarwal, Datar, Rajagopalan, Ruhl '04, Demetrescu, Finocchi, Ribichini '05]

Matching

- A matching - set of edges with no two edges sharing an end point.
- Problems:
  - Find the matching of maximum cardinality (MCM)
  - Find the matching of maximum weight (MWM)
- (Non-streamable) Algorithms:
  - Exact polytime algorithm for both [Gabow '90]
  - Linear-time $1+\epsilon$ approx for MCM [Kalantari & Shokoufandeh '95]
  - Linear-time $3/2+\epsilon$ approx for MWM [Drake & Hougardy '03]

Results

- Unweighted Matchings:

  $1+\epsilon$ approximation in constant passes.

- Weighted Matchings:

  $3+2\sqrt{2}$ approximation in single pass.

  $2+\epsilon$ approximation in constant passes.

Unweighted Matchings.

An Easy 2 Approximation

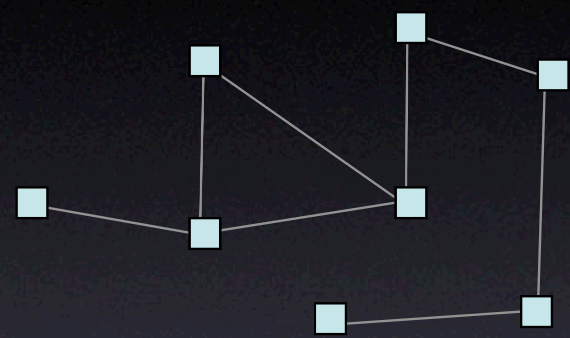
- Greedy Algorithm:

  Store an edge if it is not adjacent to stored edge

- Construct a maximal matching - 2 Approximation

Augmenting Paths

Augmenting Paths



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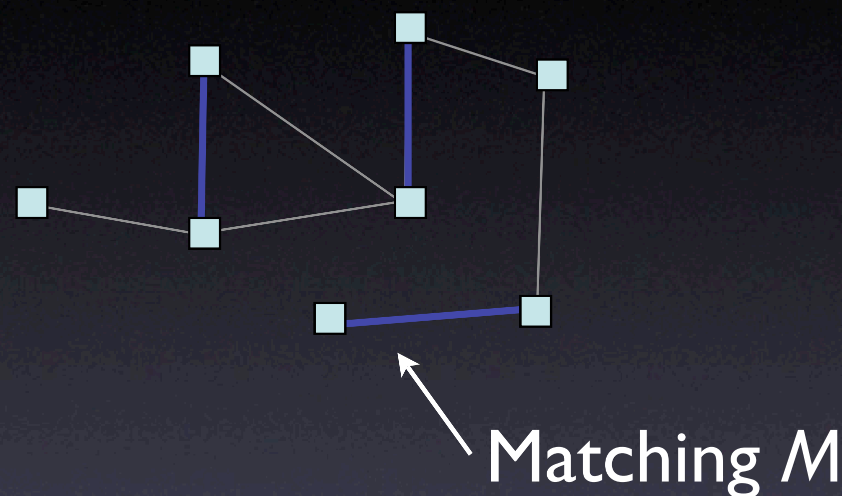

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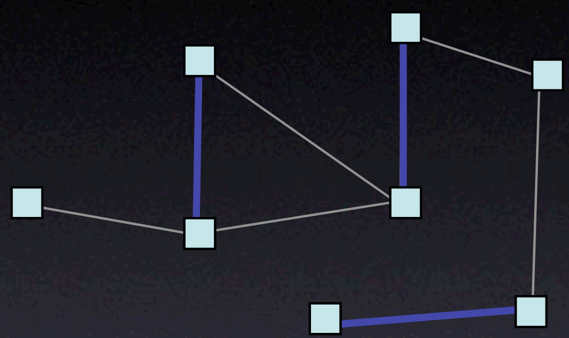

- Augmenting Path: simple path starting and ending at unmatched nodes such that edges alternate between M and $E \setminus M$.

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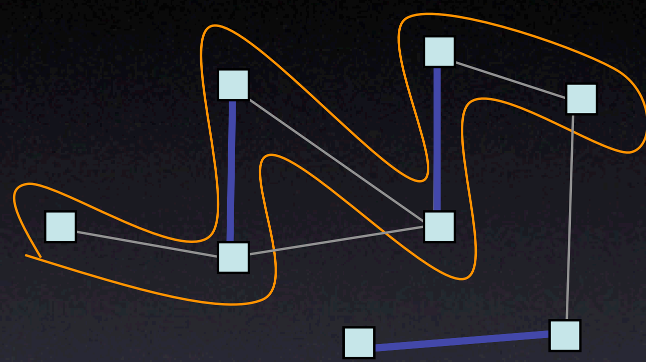

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- Augmenting Path: simple path starting and ending at unmatched nodes such that edges alternate between *M* and *E**M*.

Augmenting Paths

- Consider augmenting paths defined by taking the symmetric difference between current (maximal) matching and optimum matching.
- Let P_i be the number of length *i* augmenting paths

$$|M| + \sum_{1 \le i \le k} P_i \ge OPT(1 - 1/k)$$

Algorithm Outline

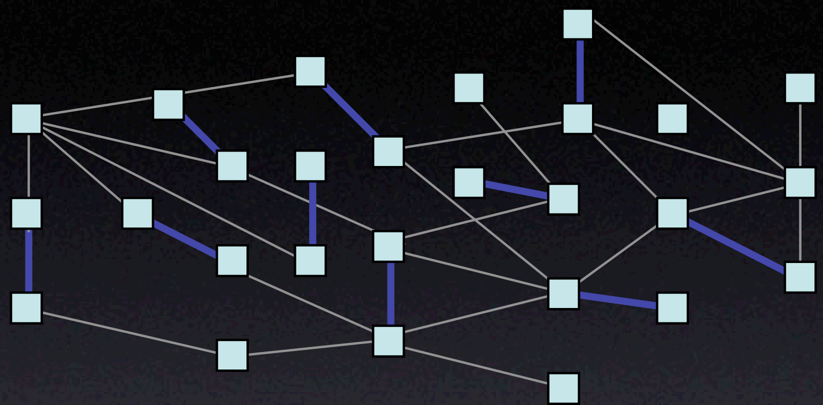
1. Find a maximal matching
2. For $1 \le i \le k$:

   Find a set, S_i, of length i augmenting paths

3. Augment current matching with S_j where j = argmax S_i
4. Repeat from 2 unless S_j is small

Projecting to Layered Graphs

G



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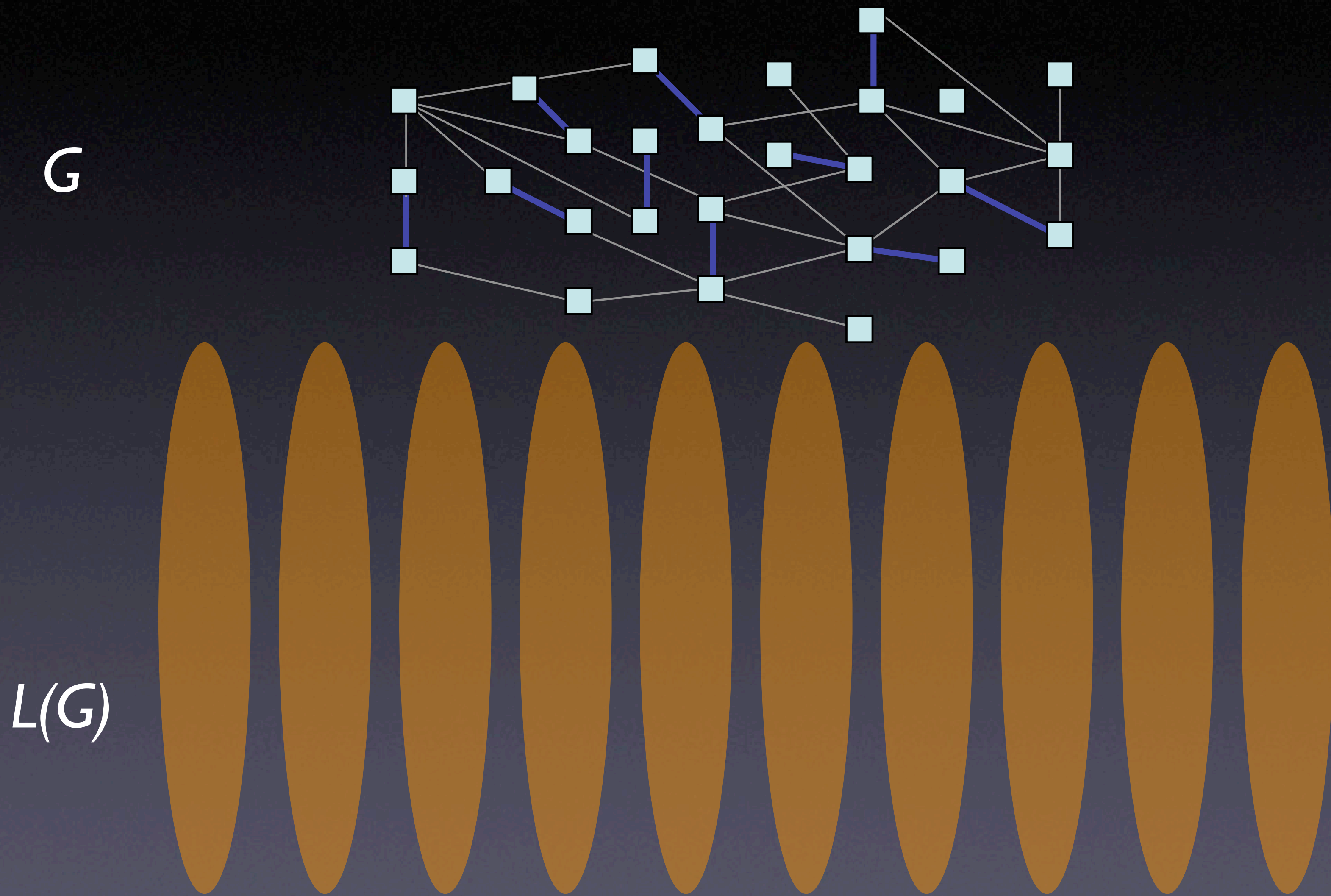

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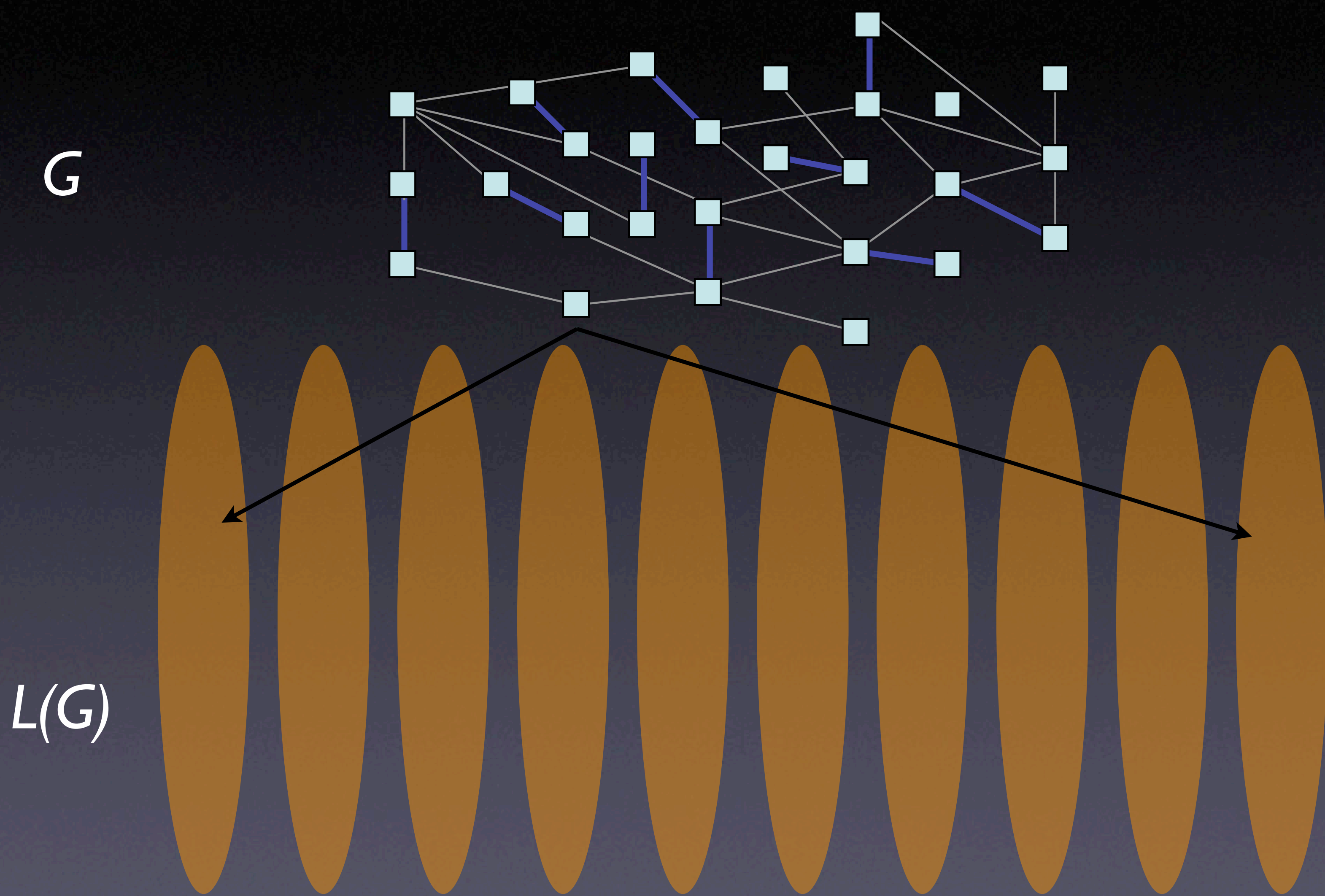

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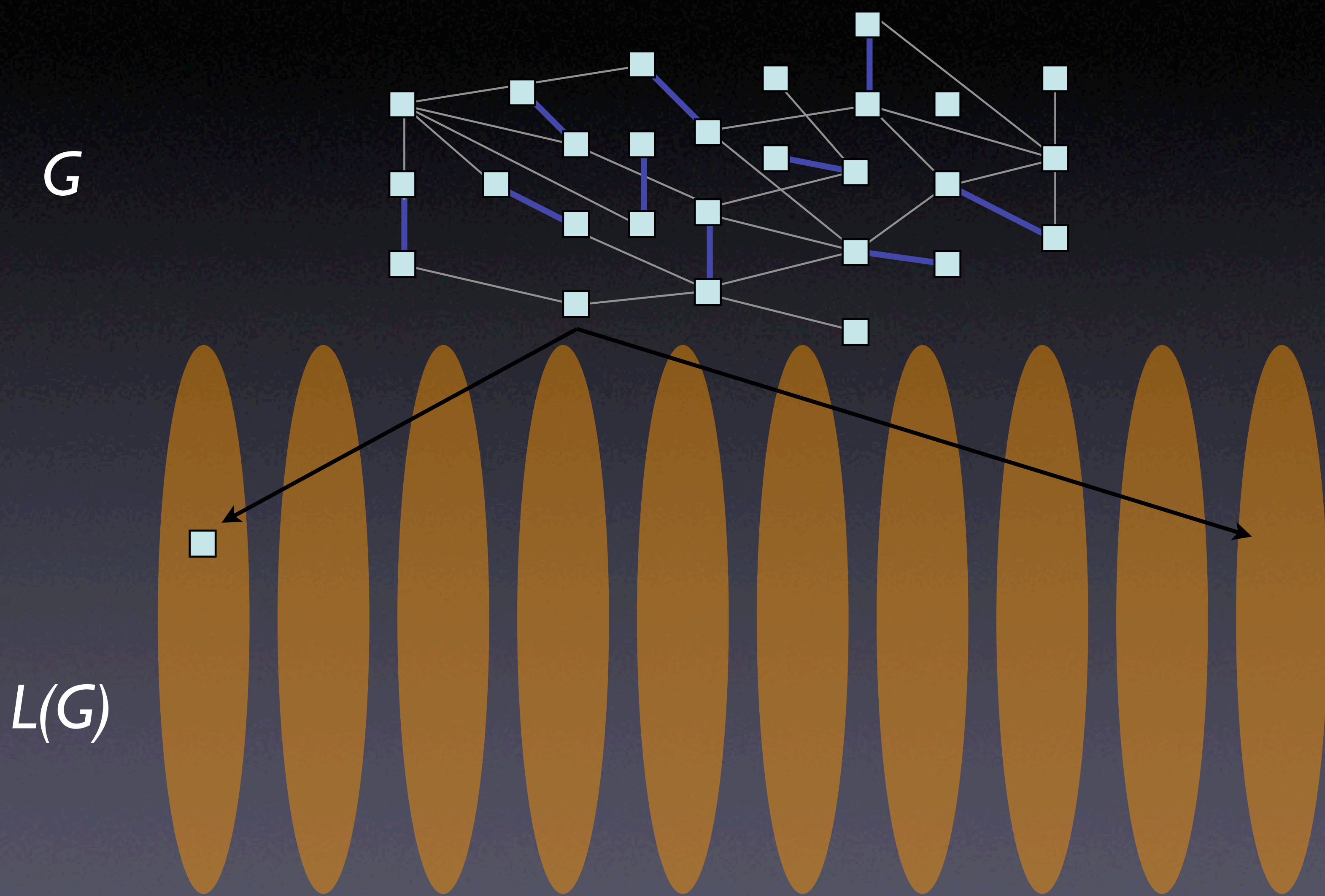

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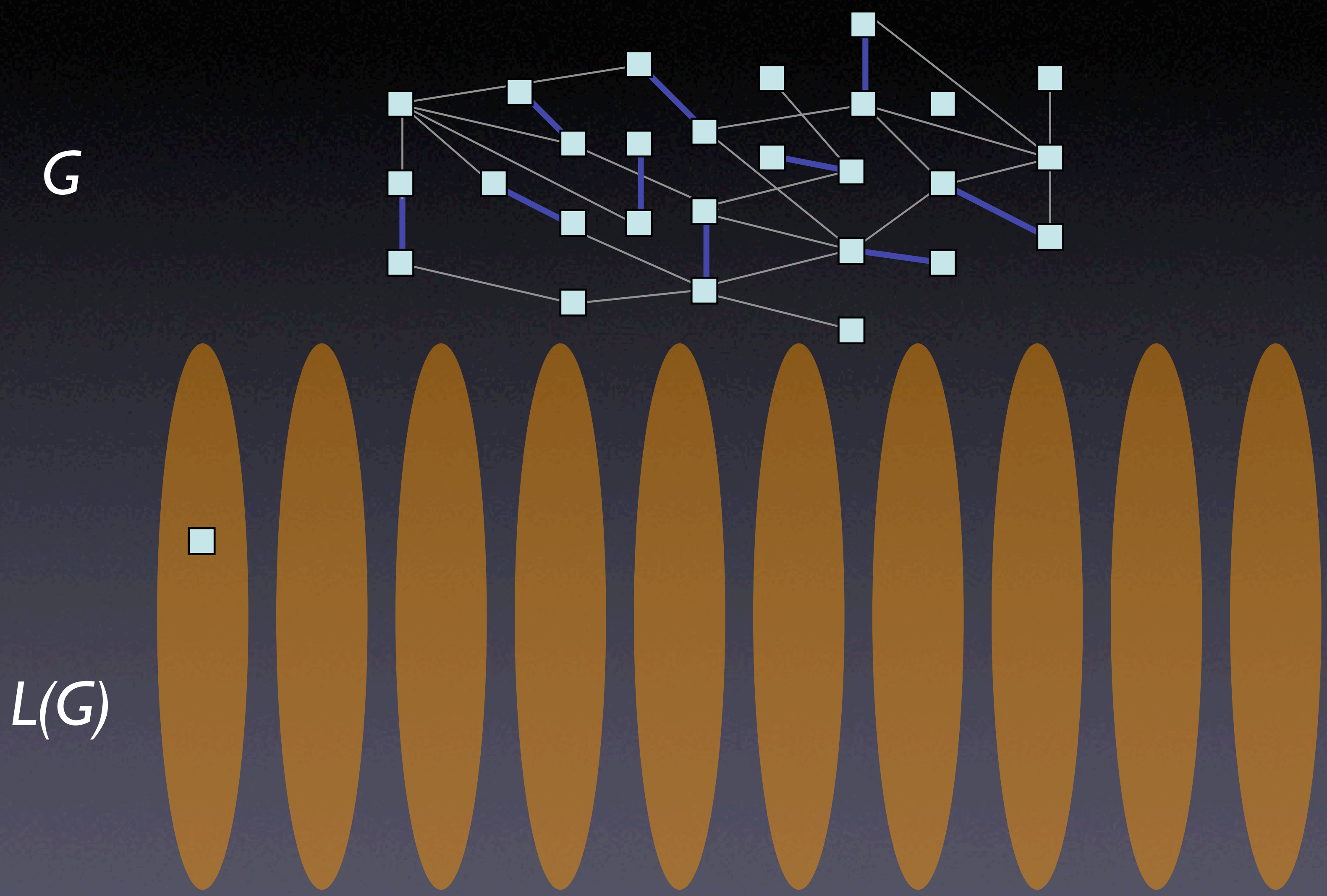

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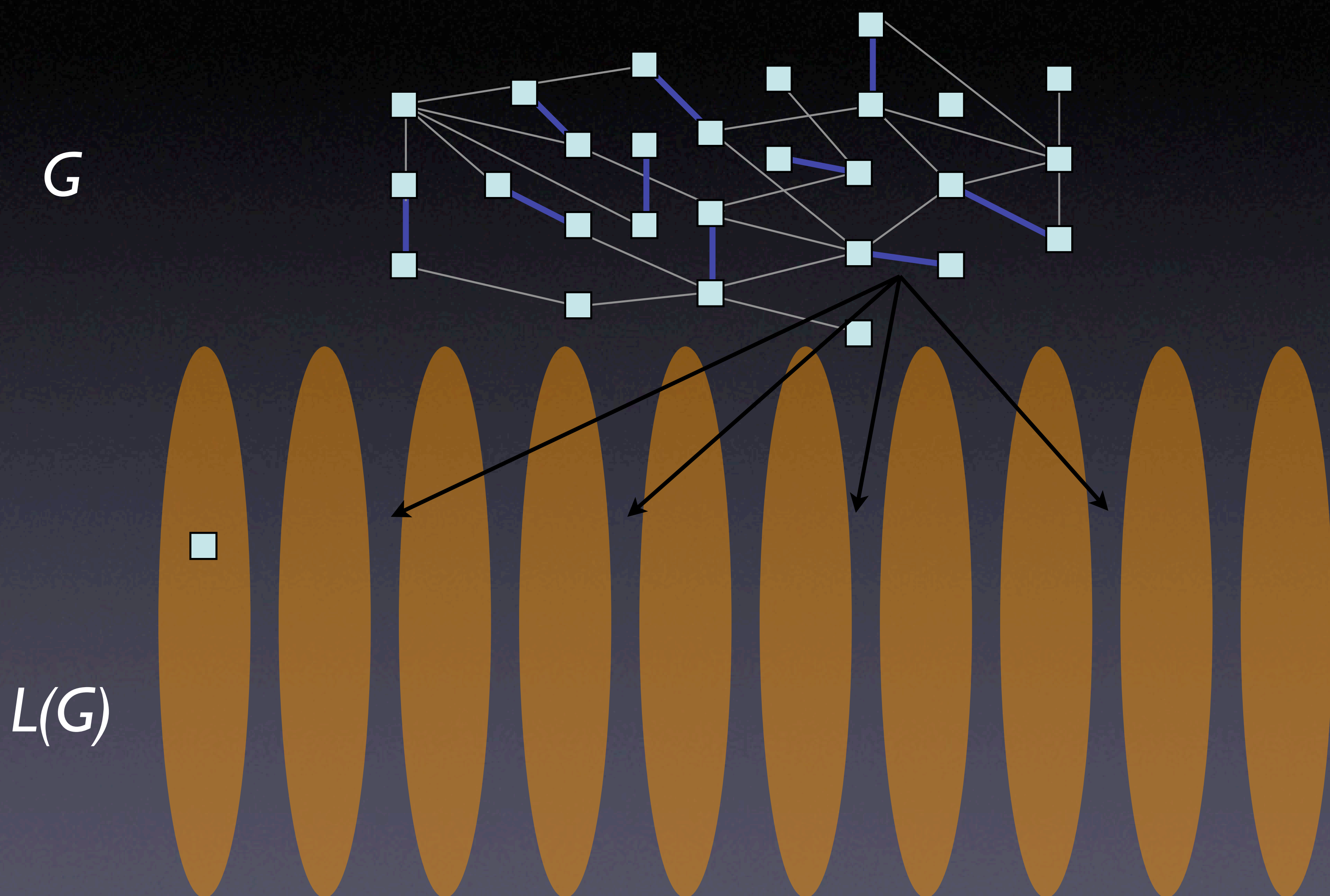

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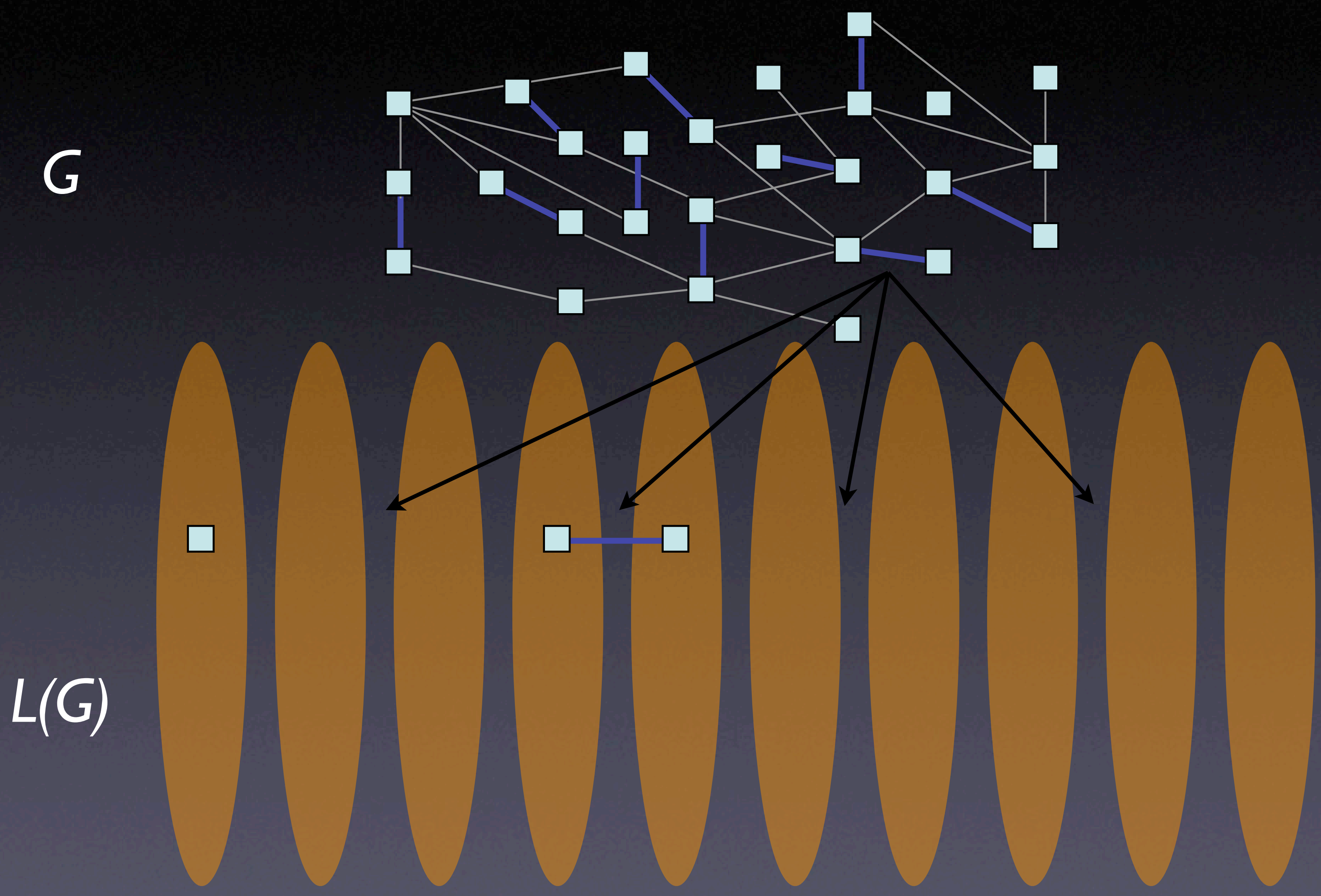

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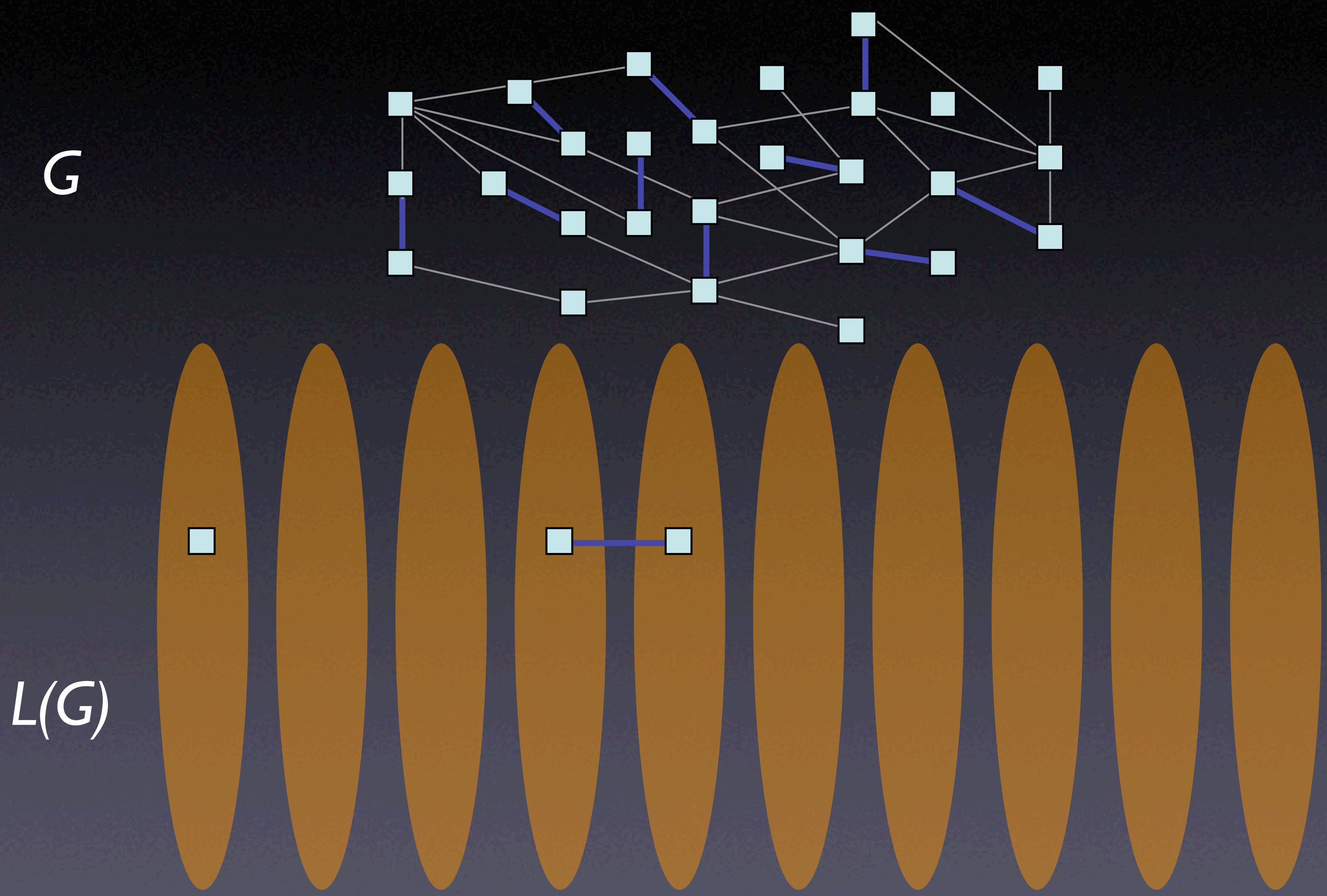

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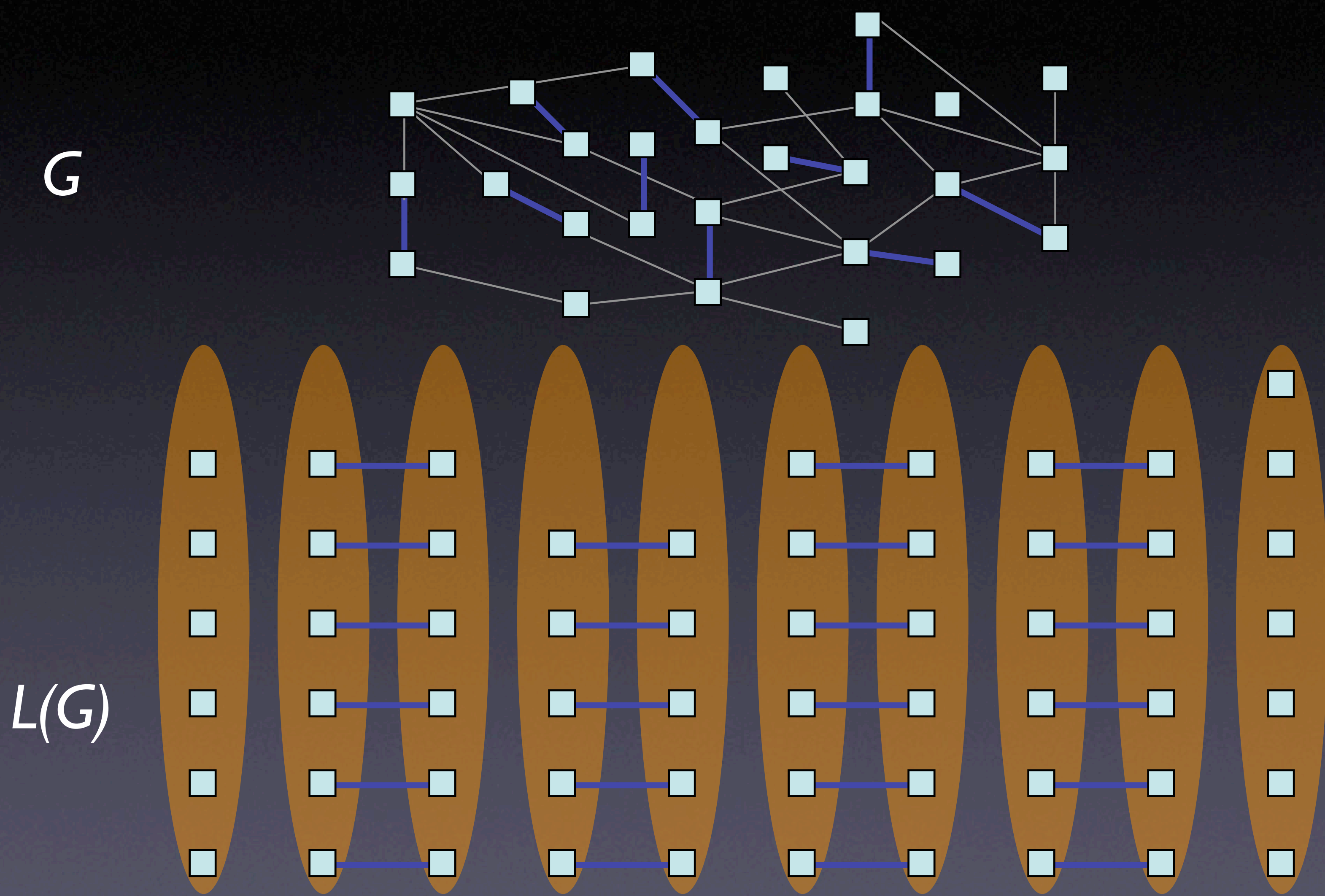

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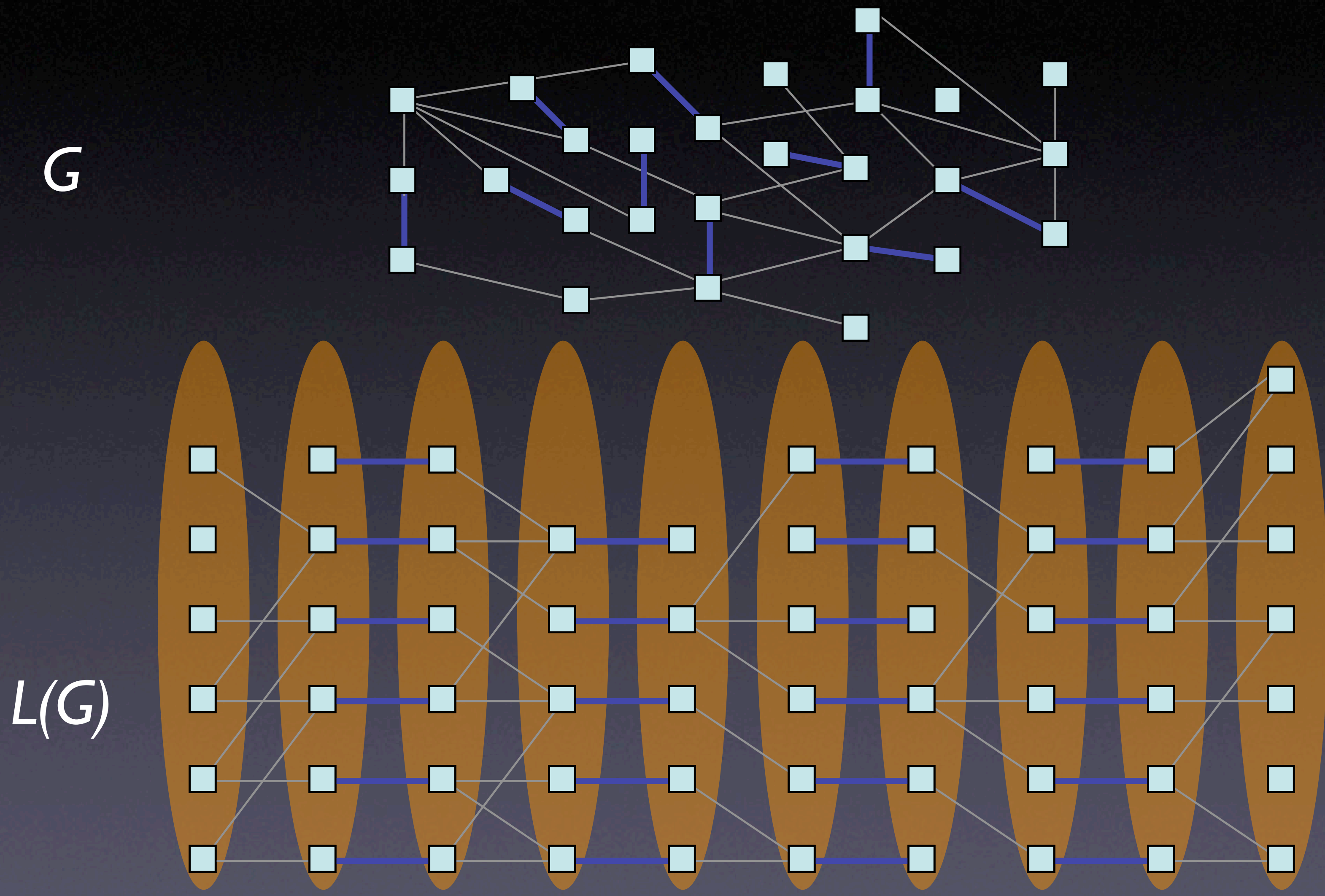

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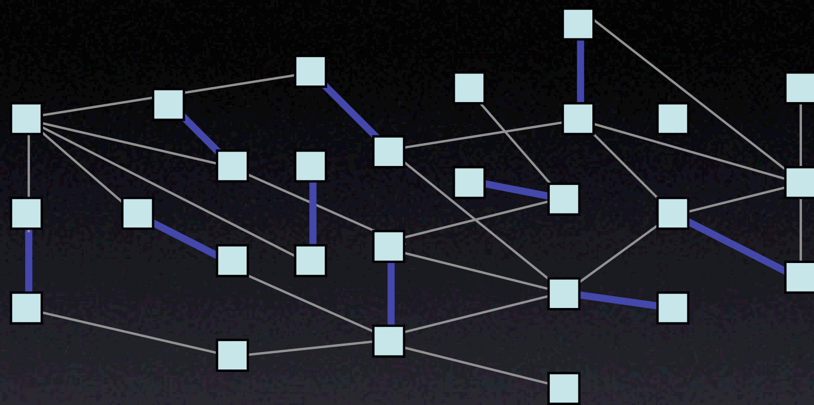
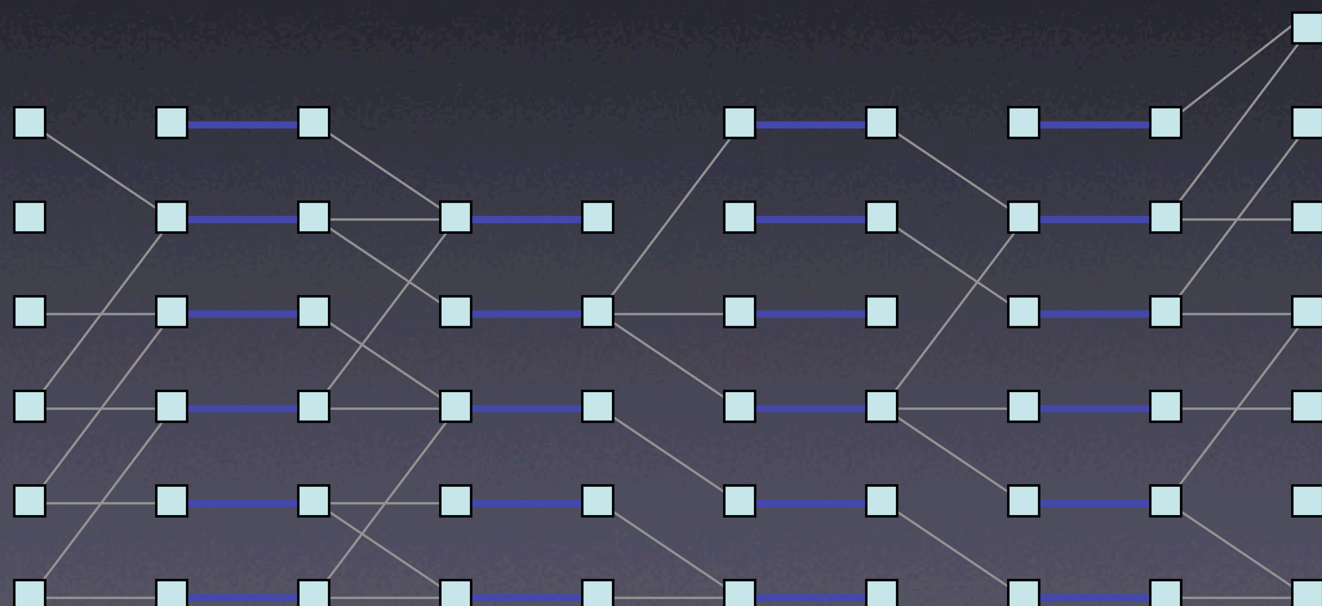


L(G)



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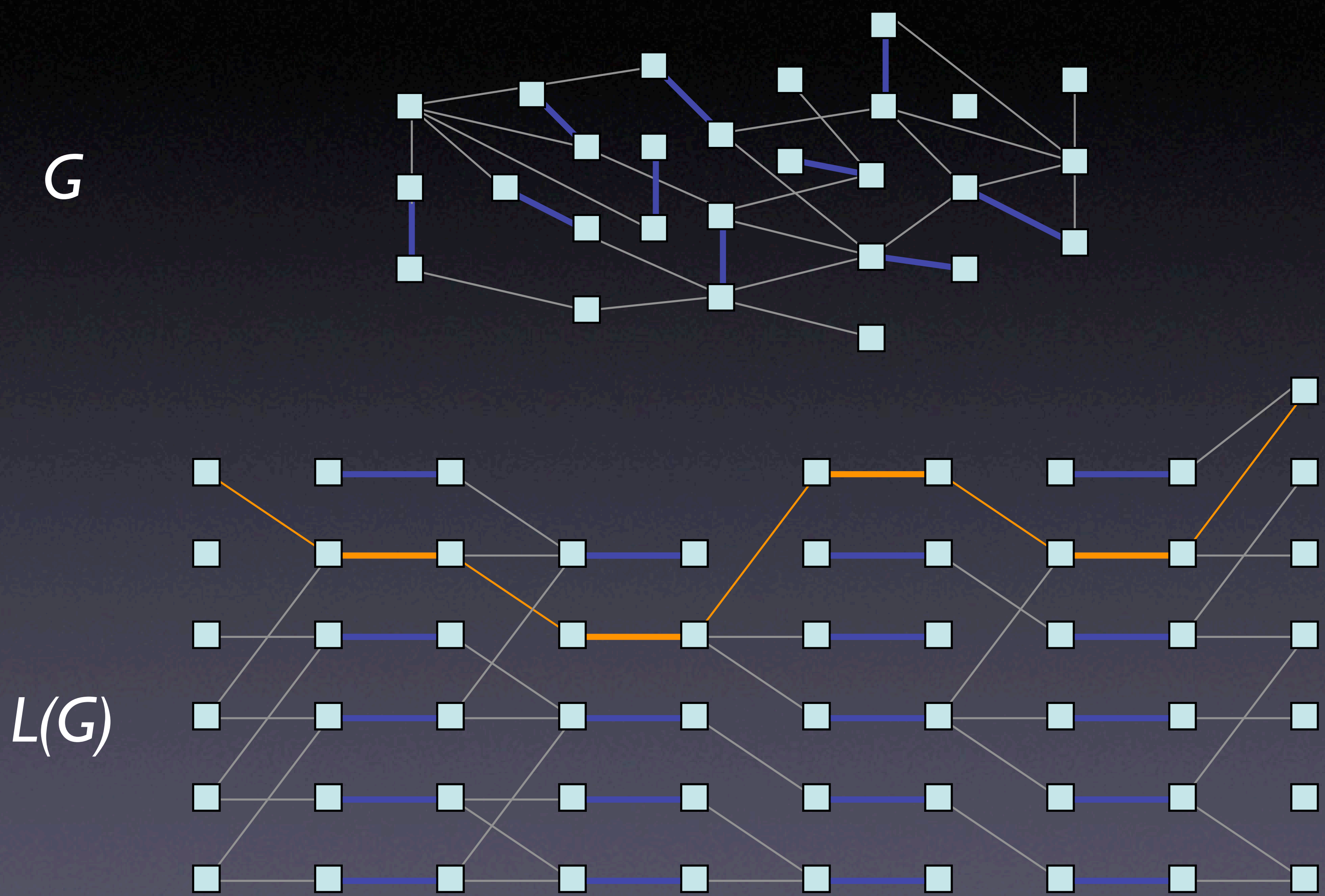

Lemma: If there are P_i length i augmenting paths in G then we expect $P_i \, / \, 2(2i)^i$ node disjoint paths in $L(G)$.

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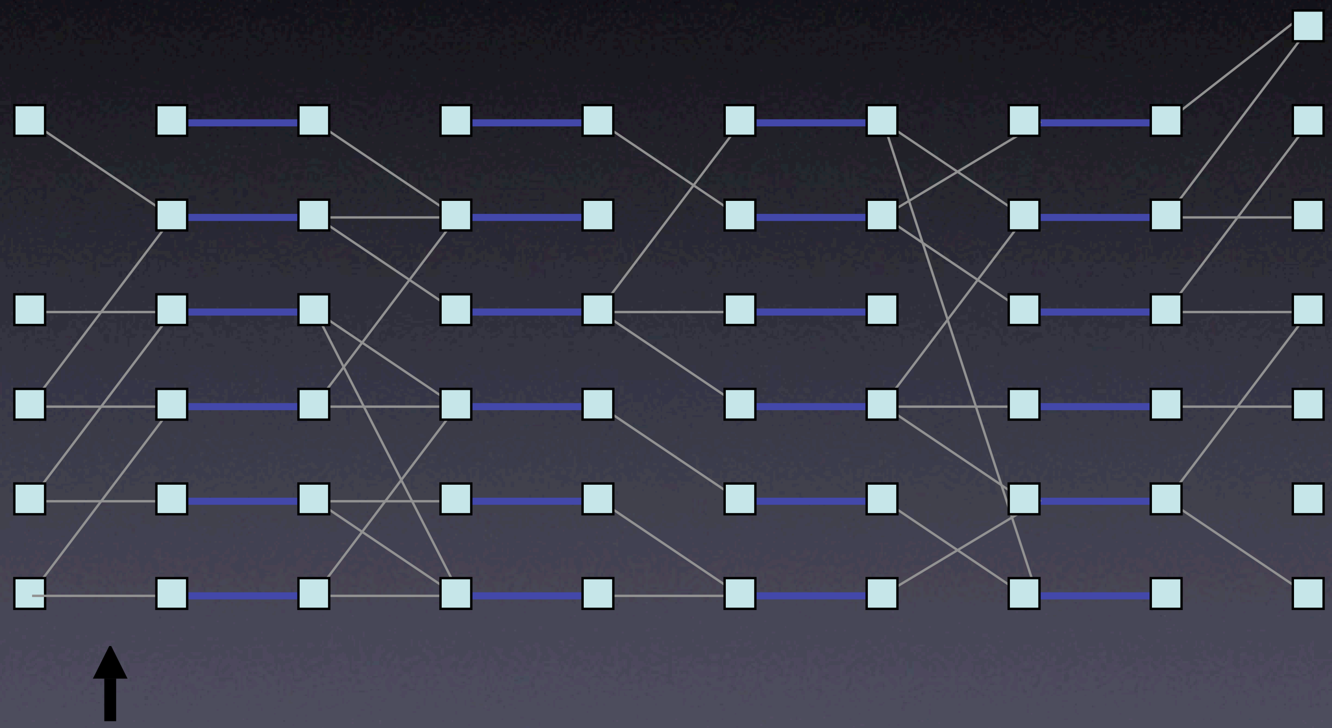
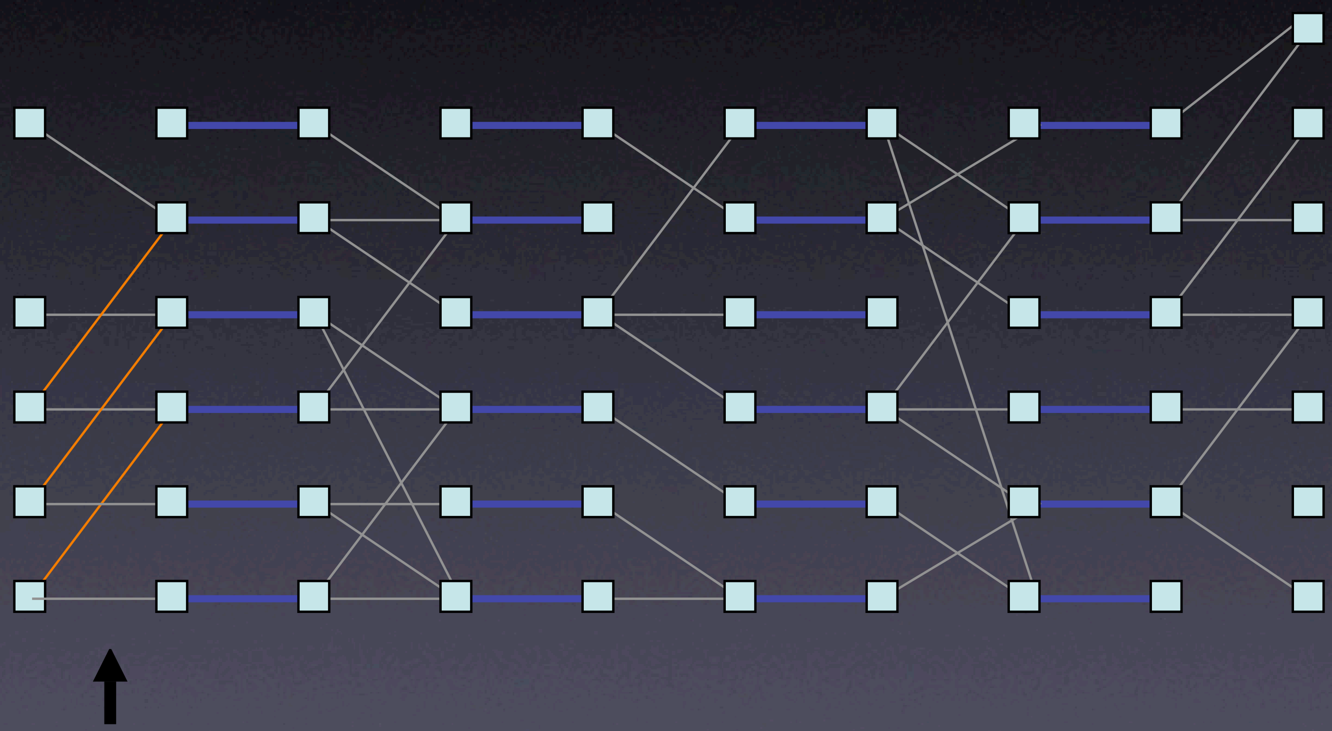
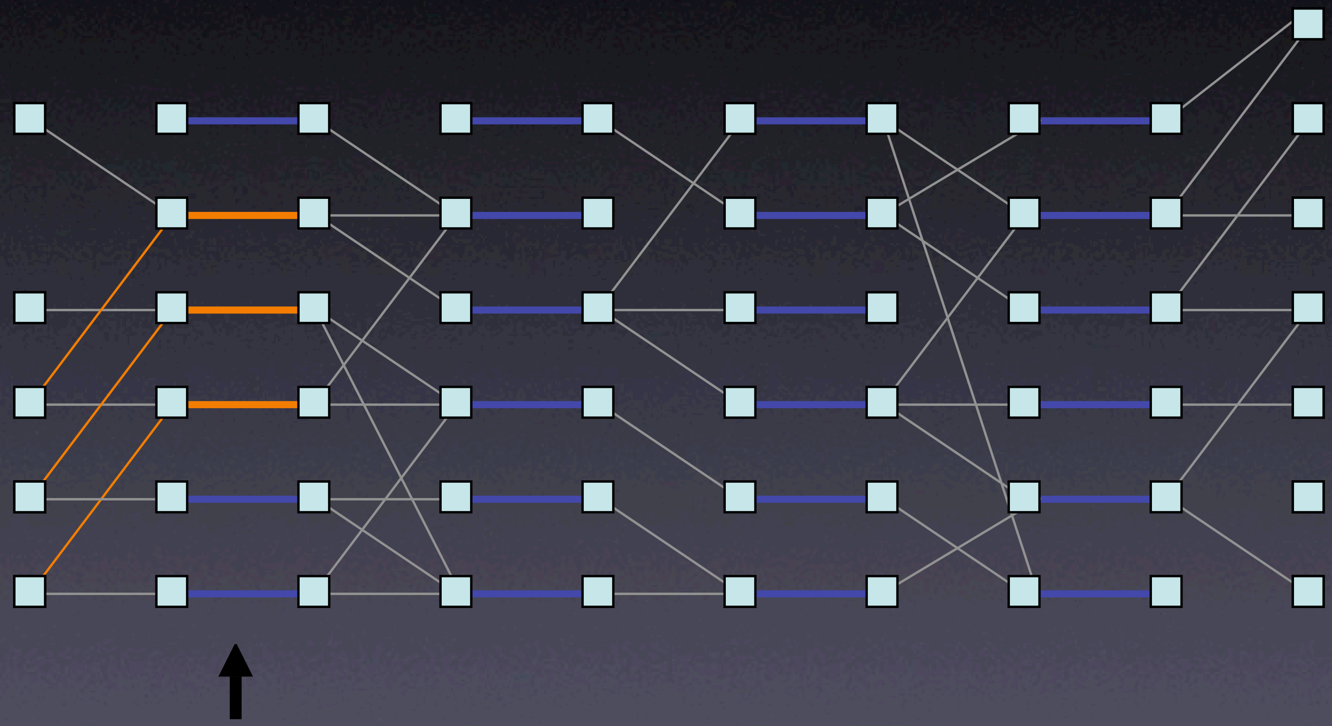
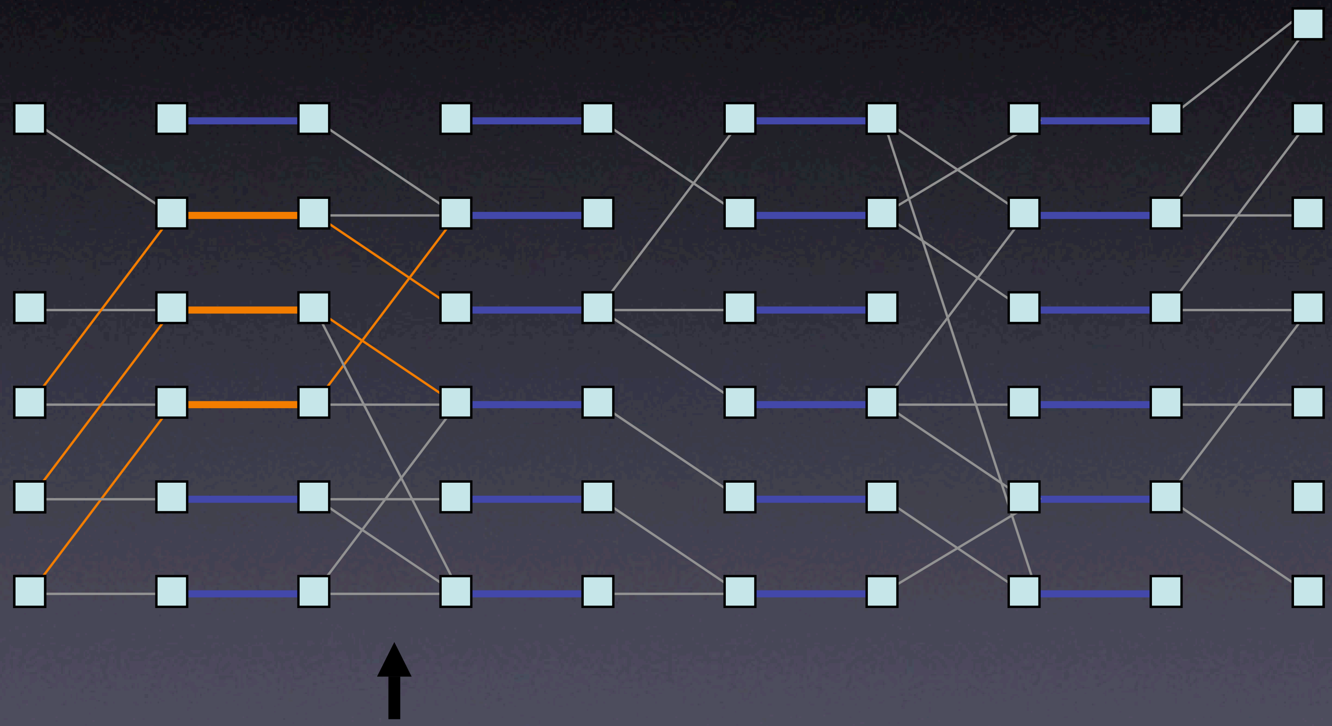
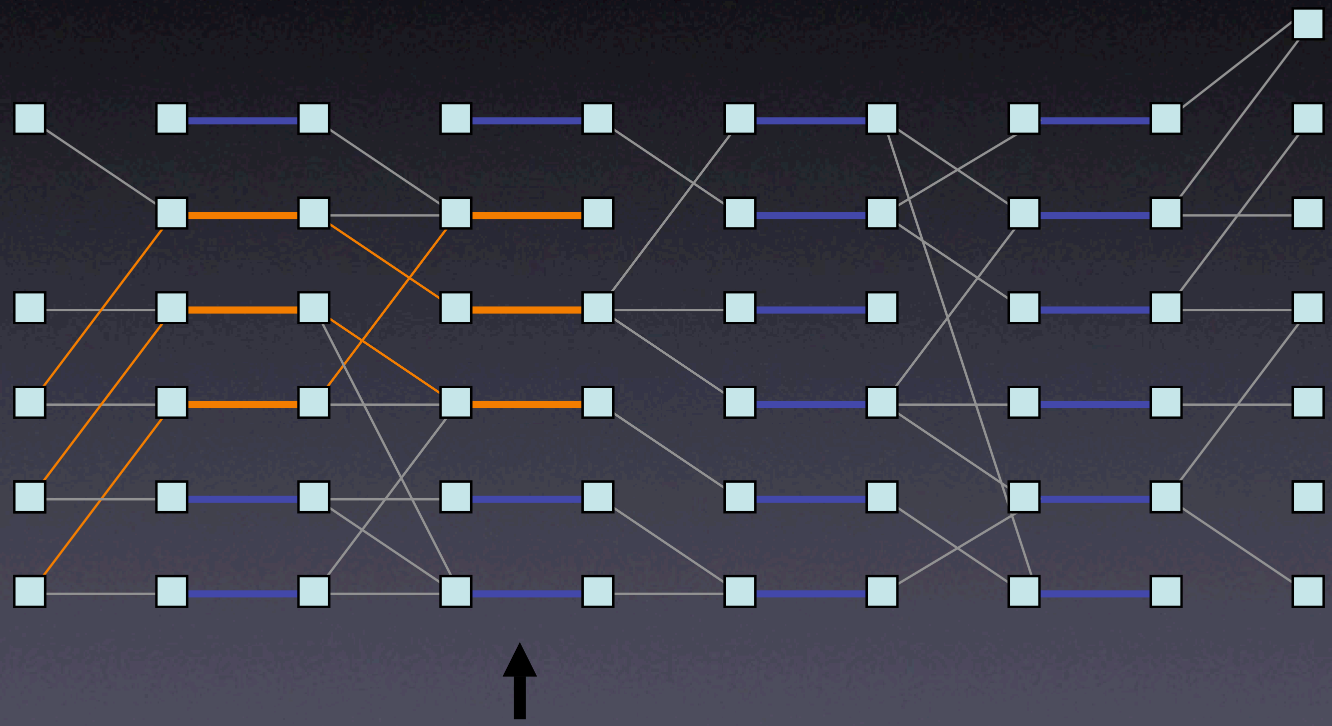
To find a constant fraction of length i augmenting paths P_i, create layered graph and greedily find node disjoint paths.

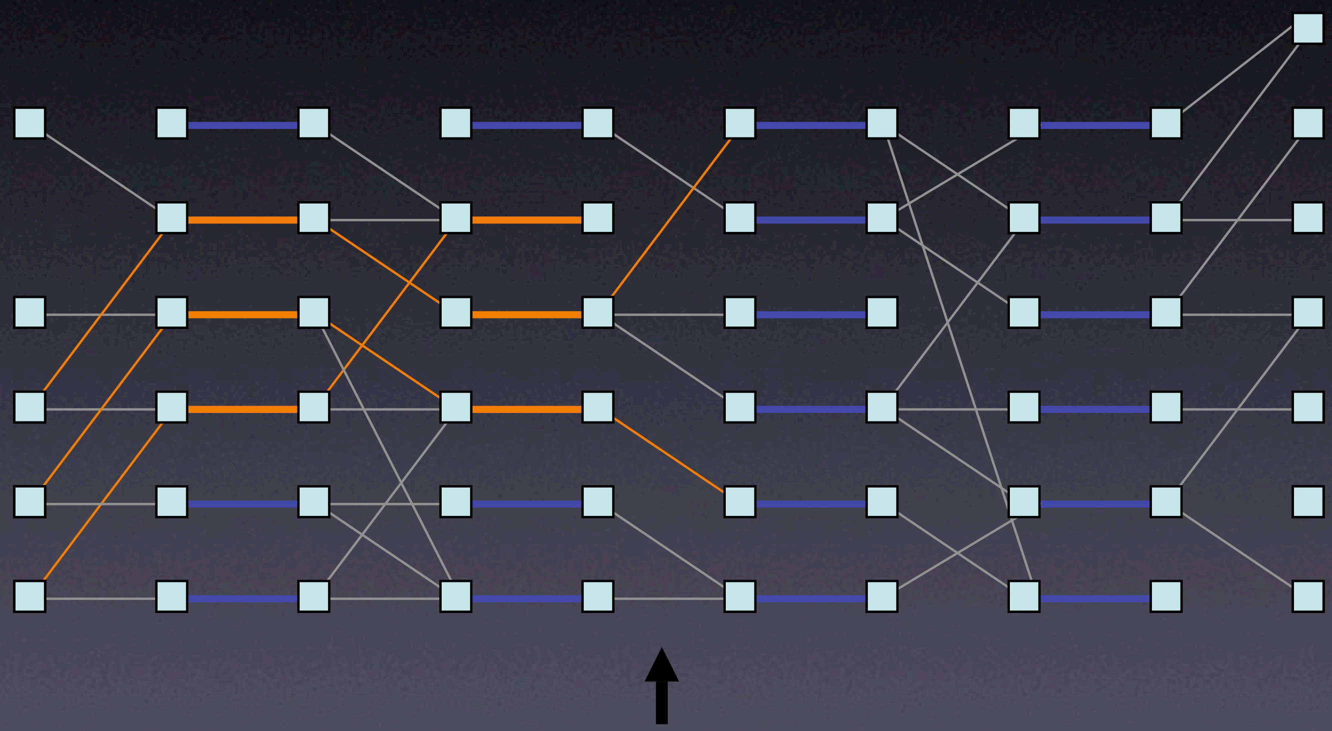


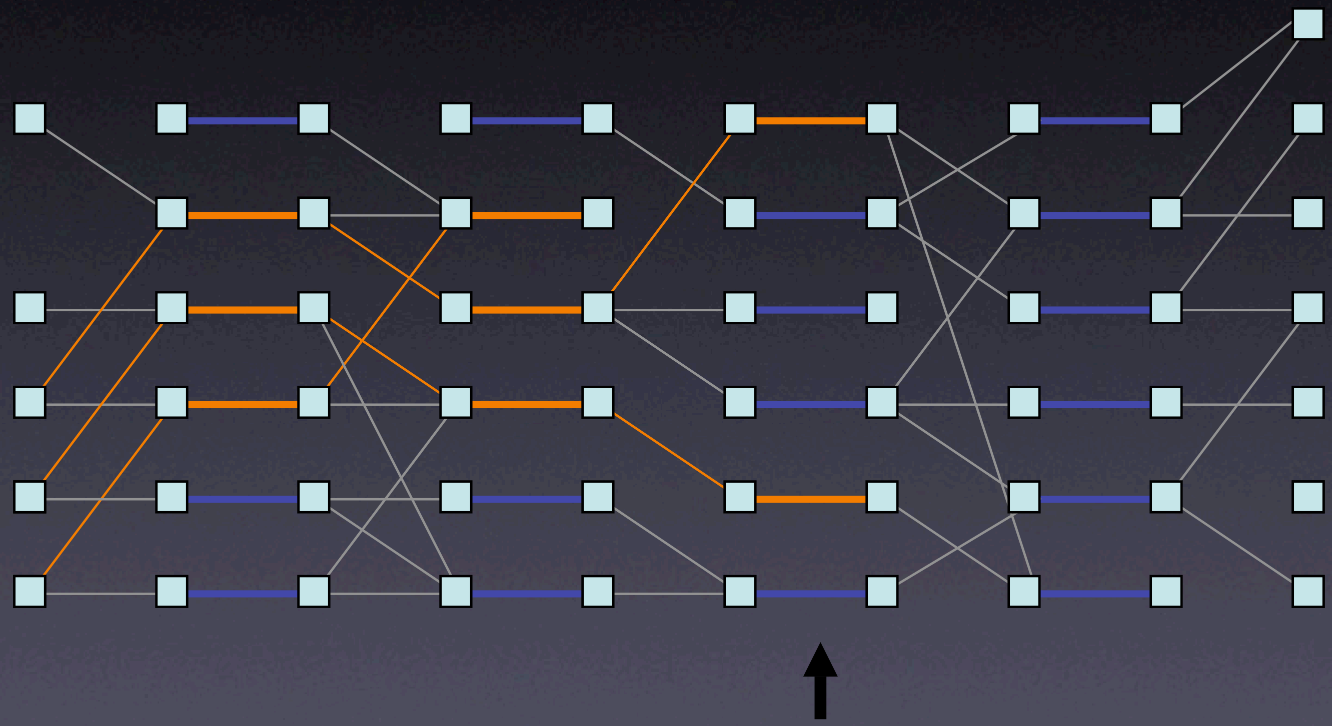


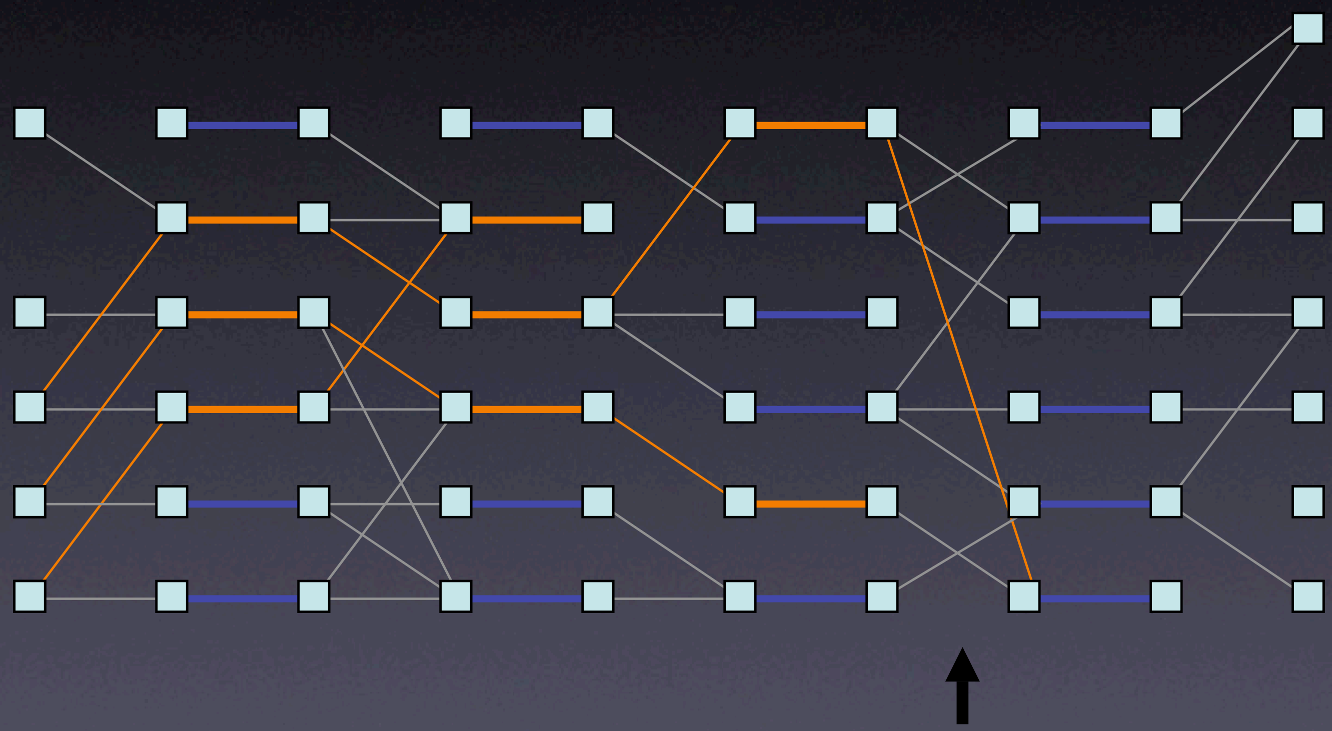


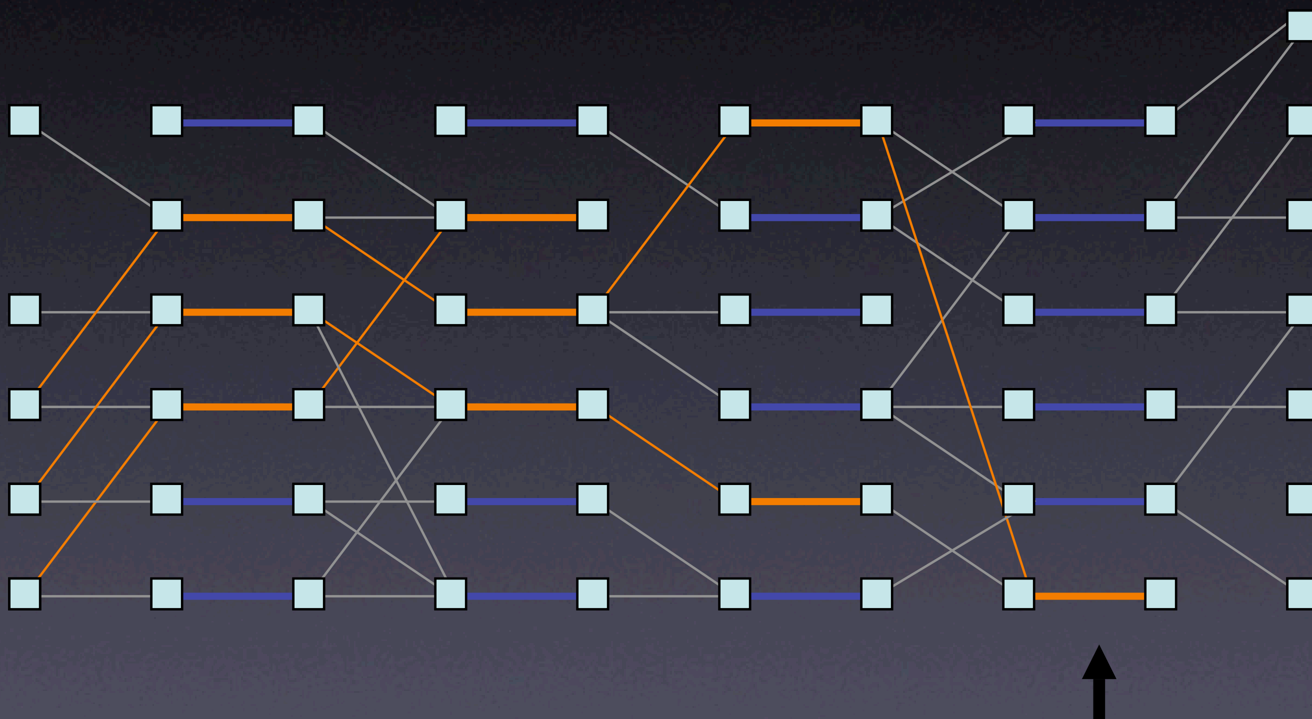


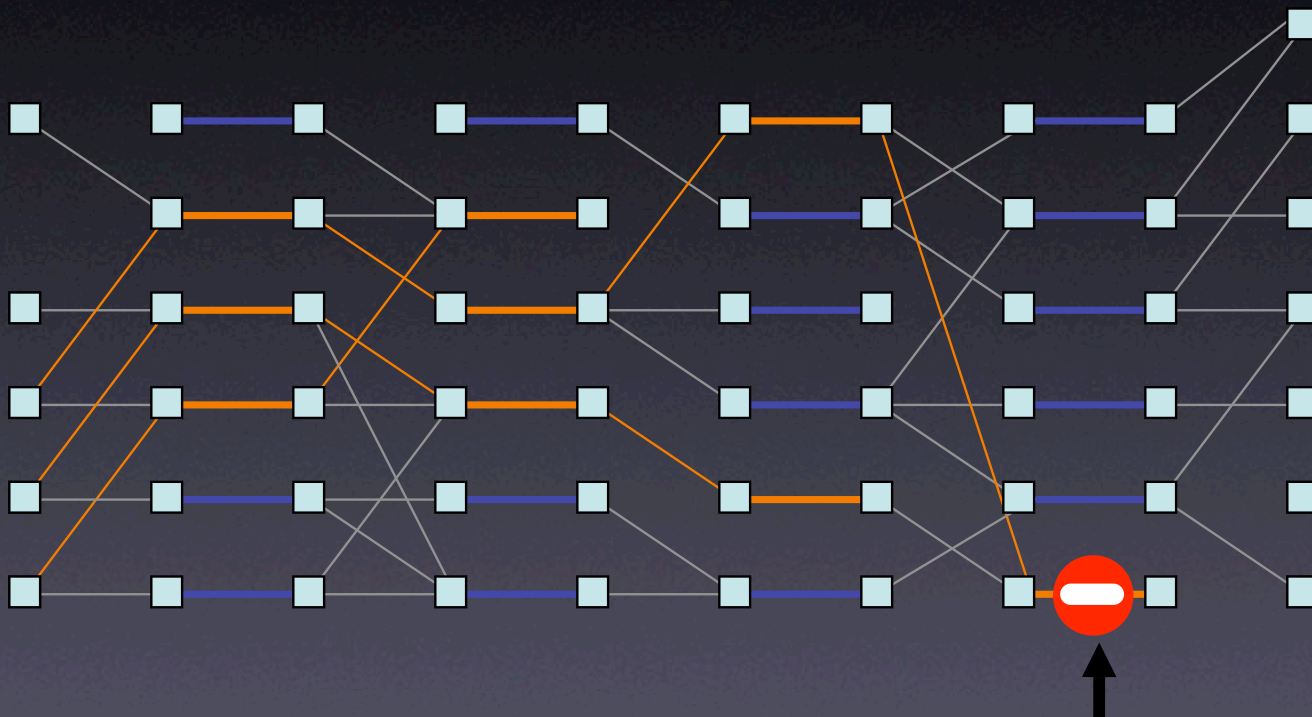


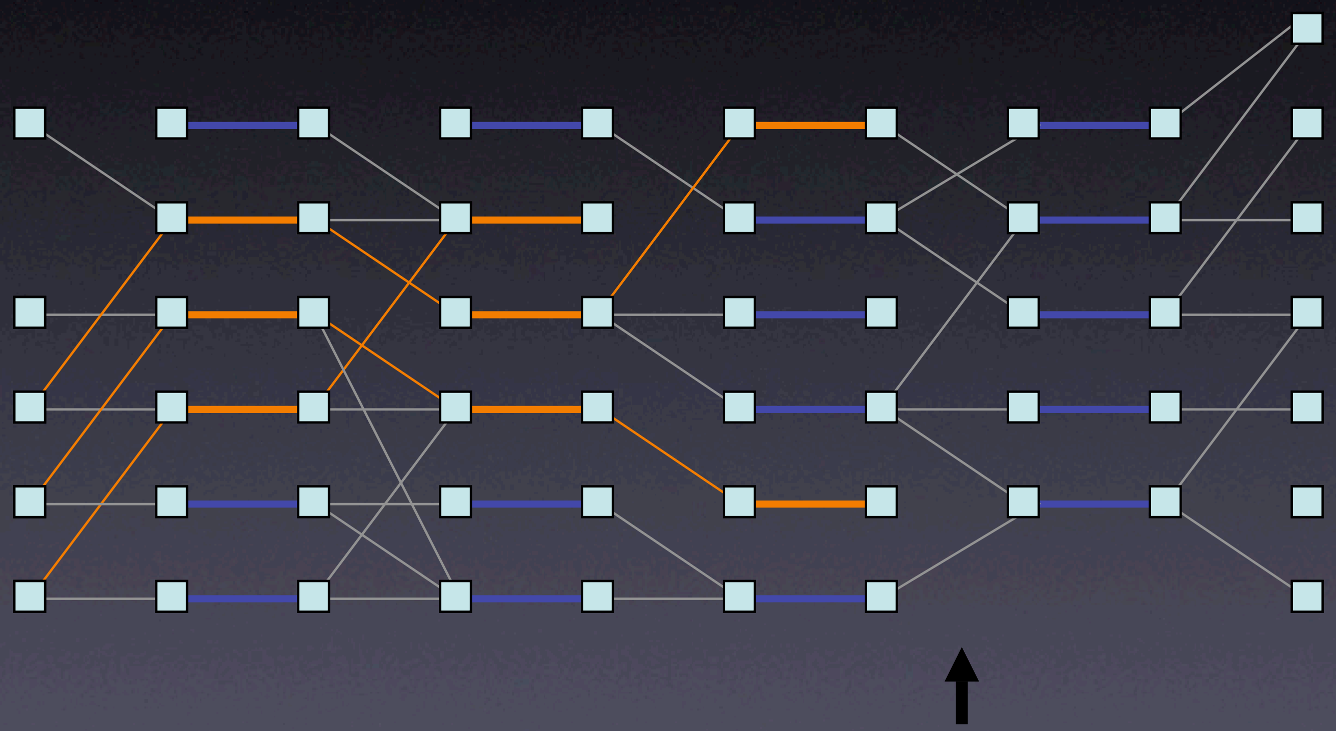


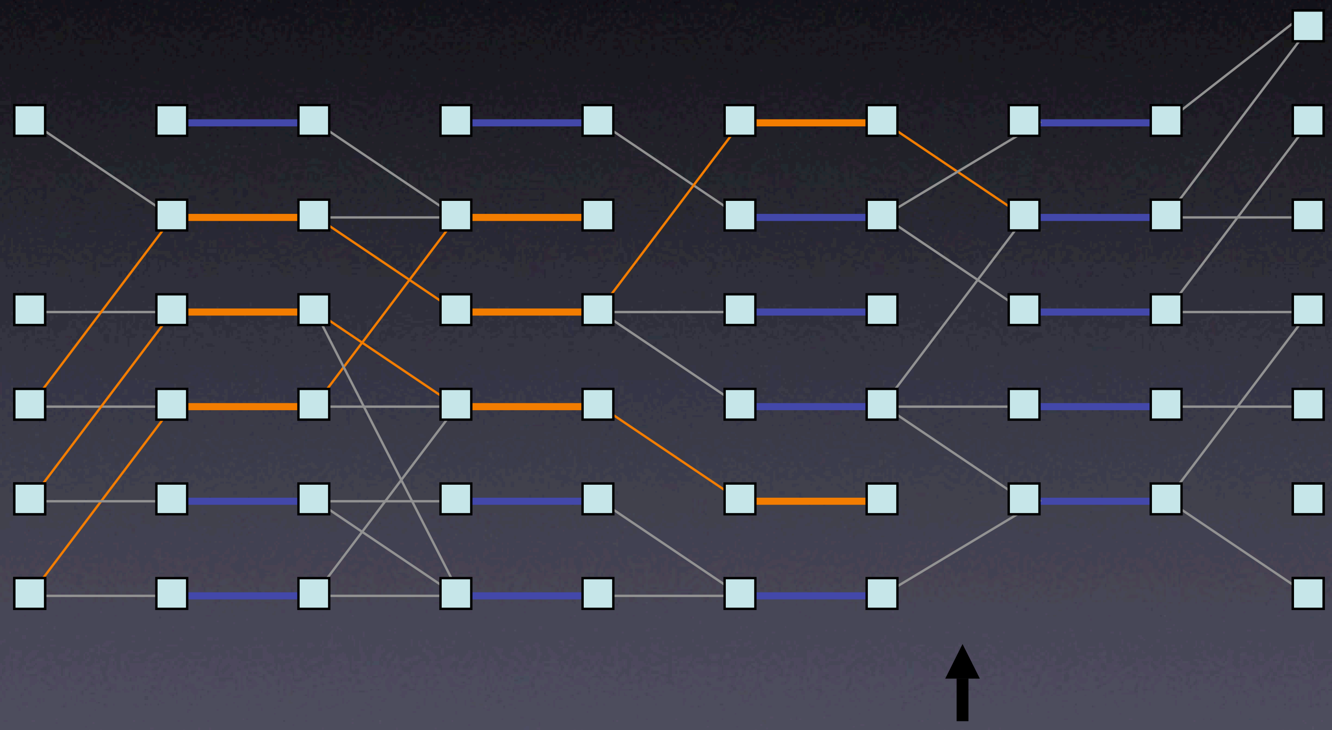


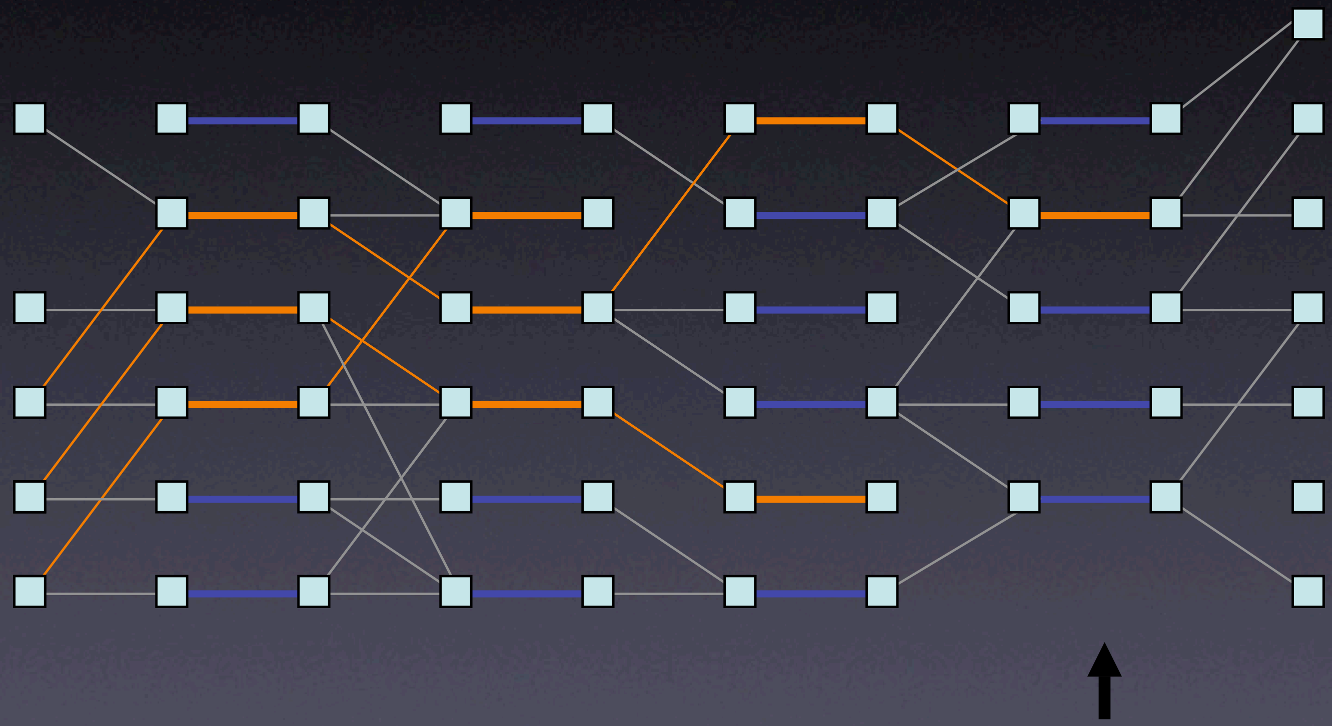


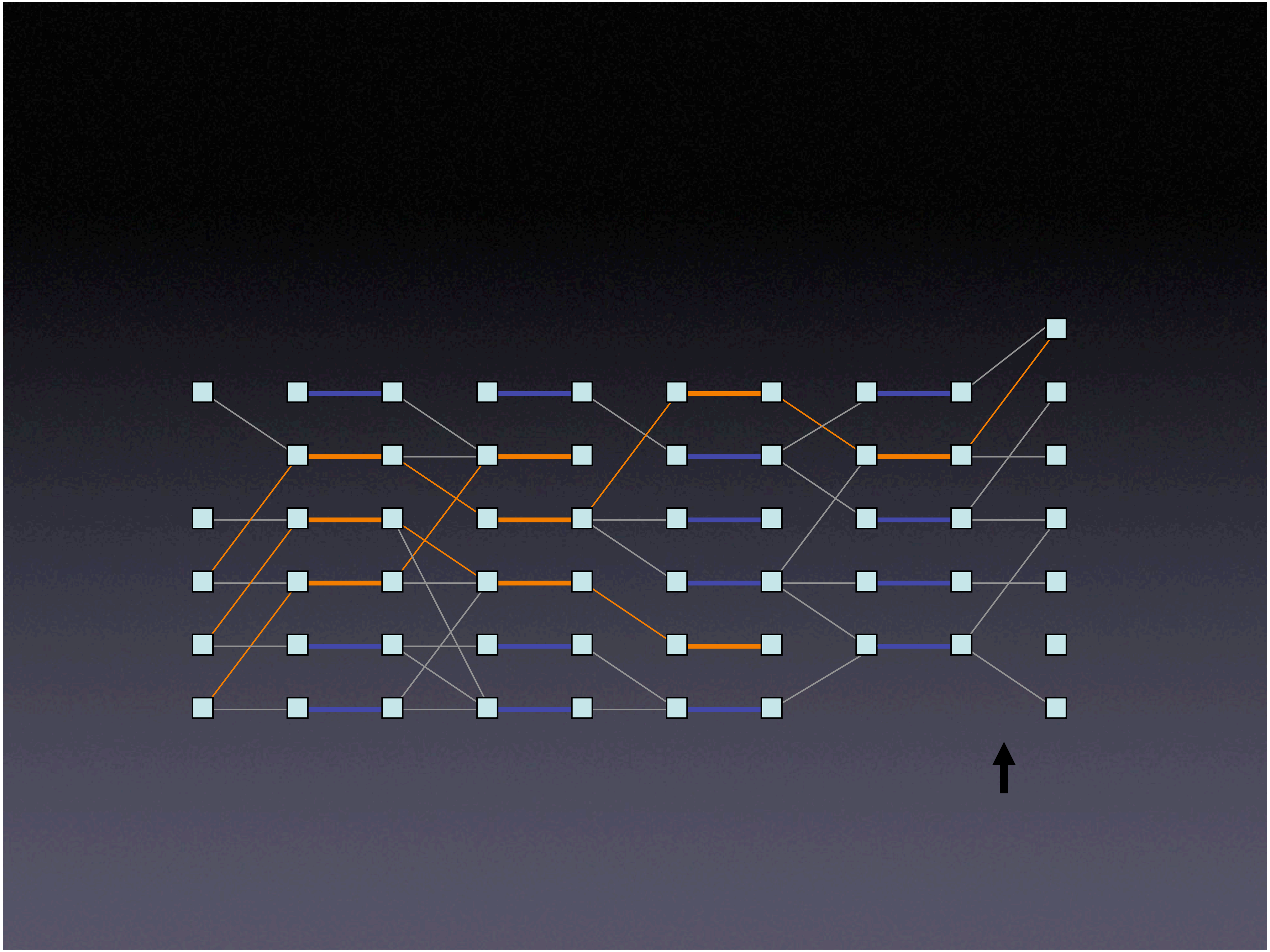


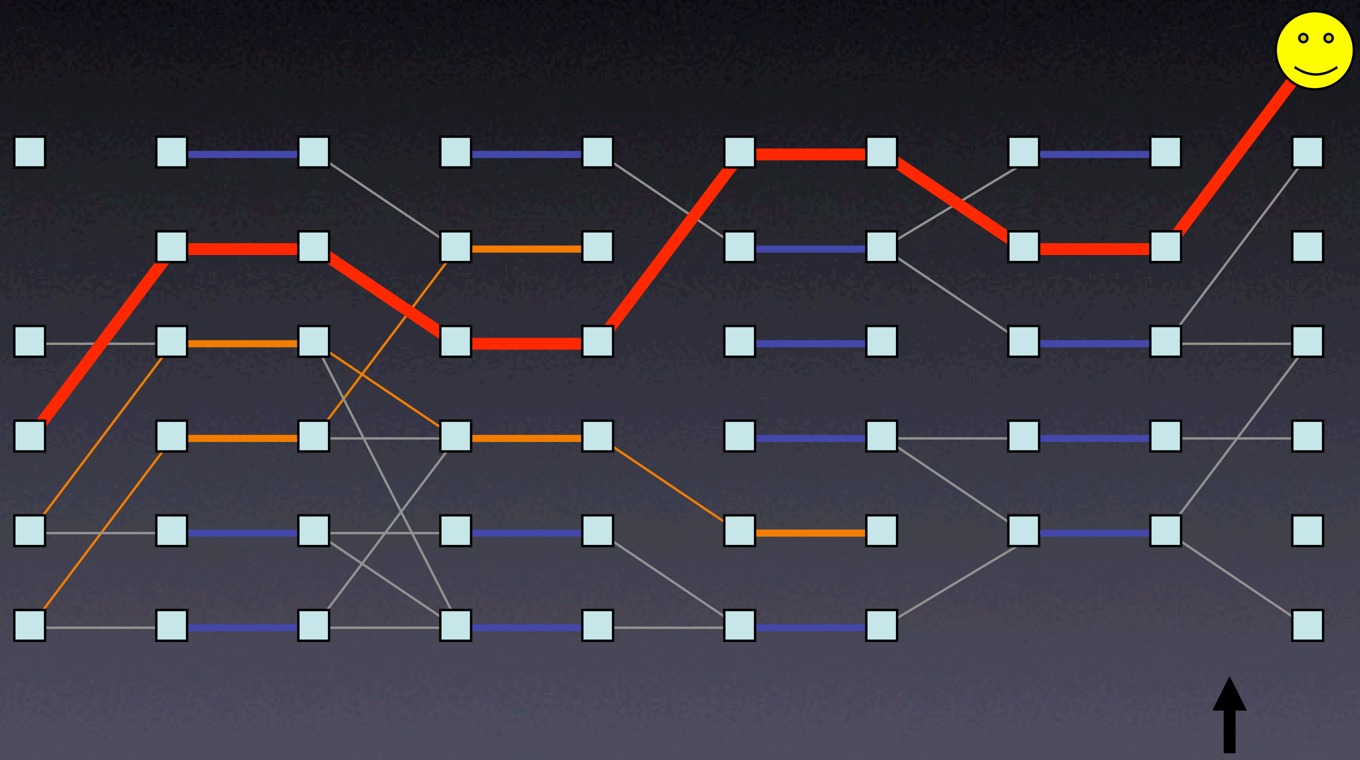


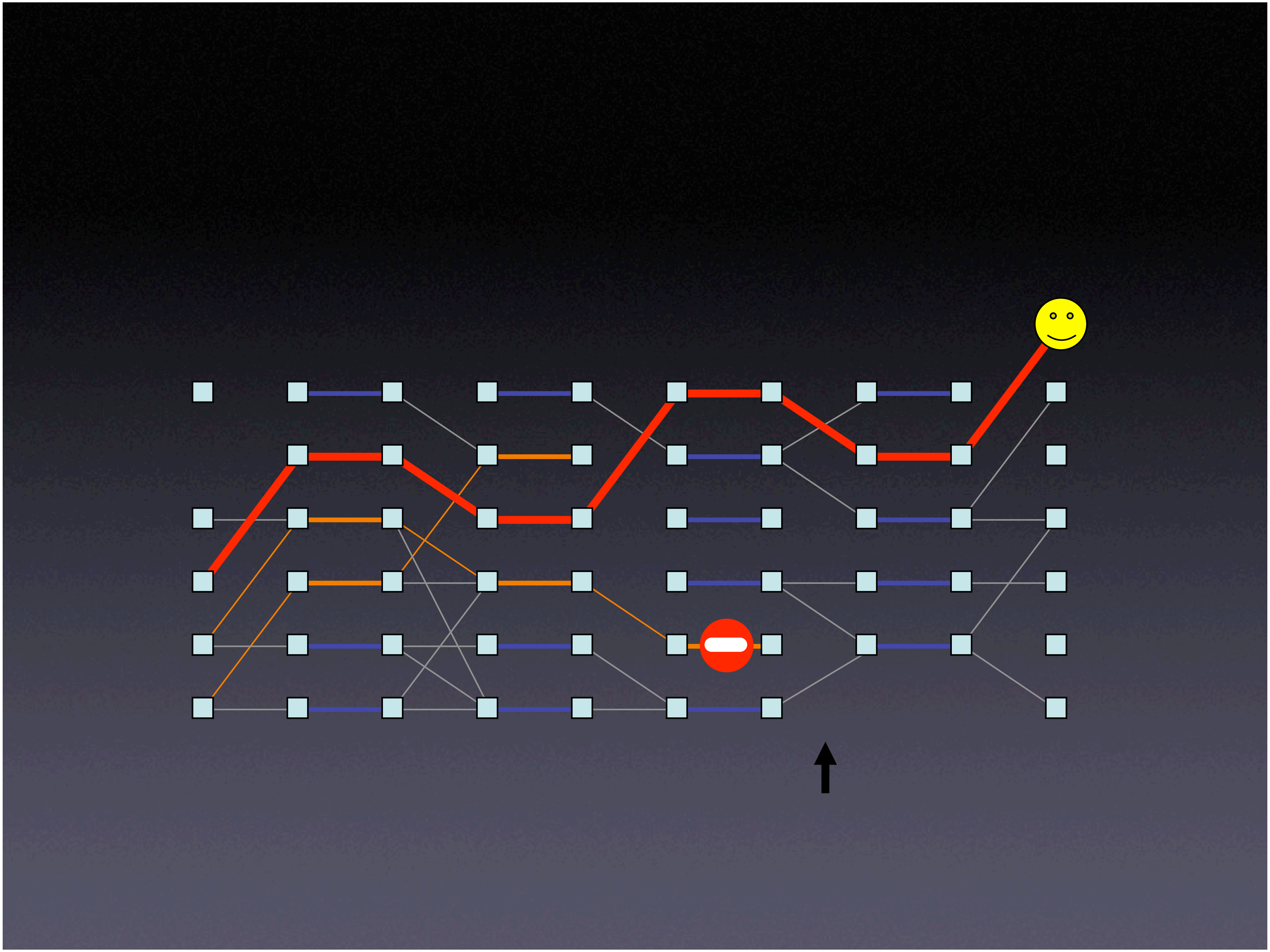


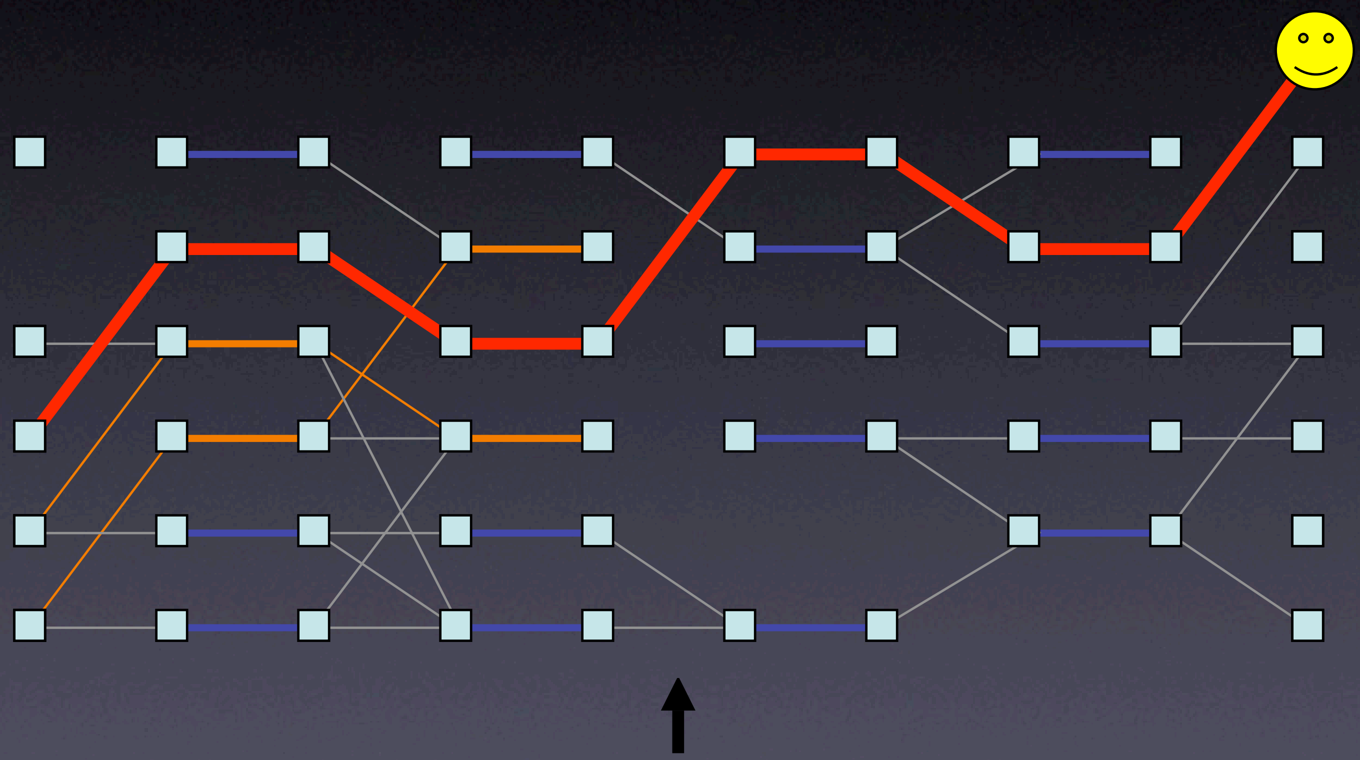


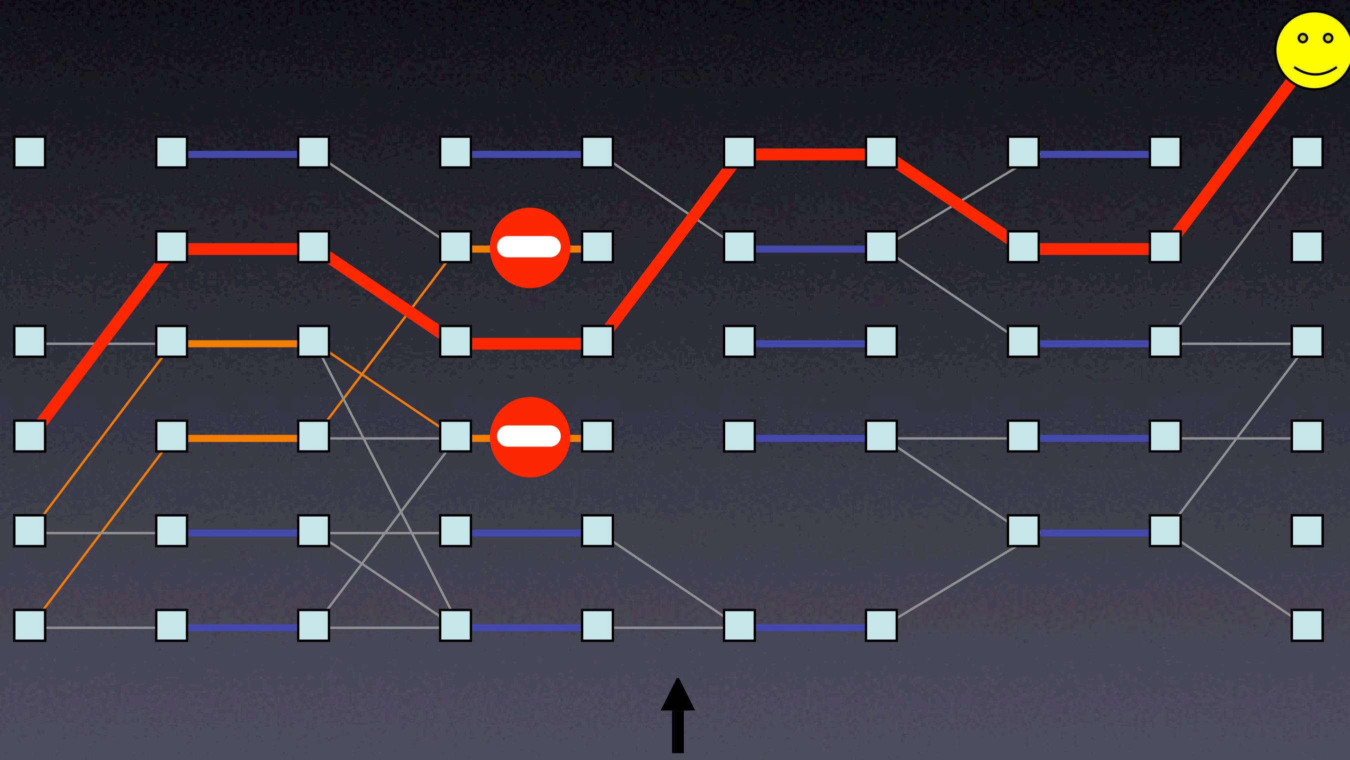


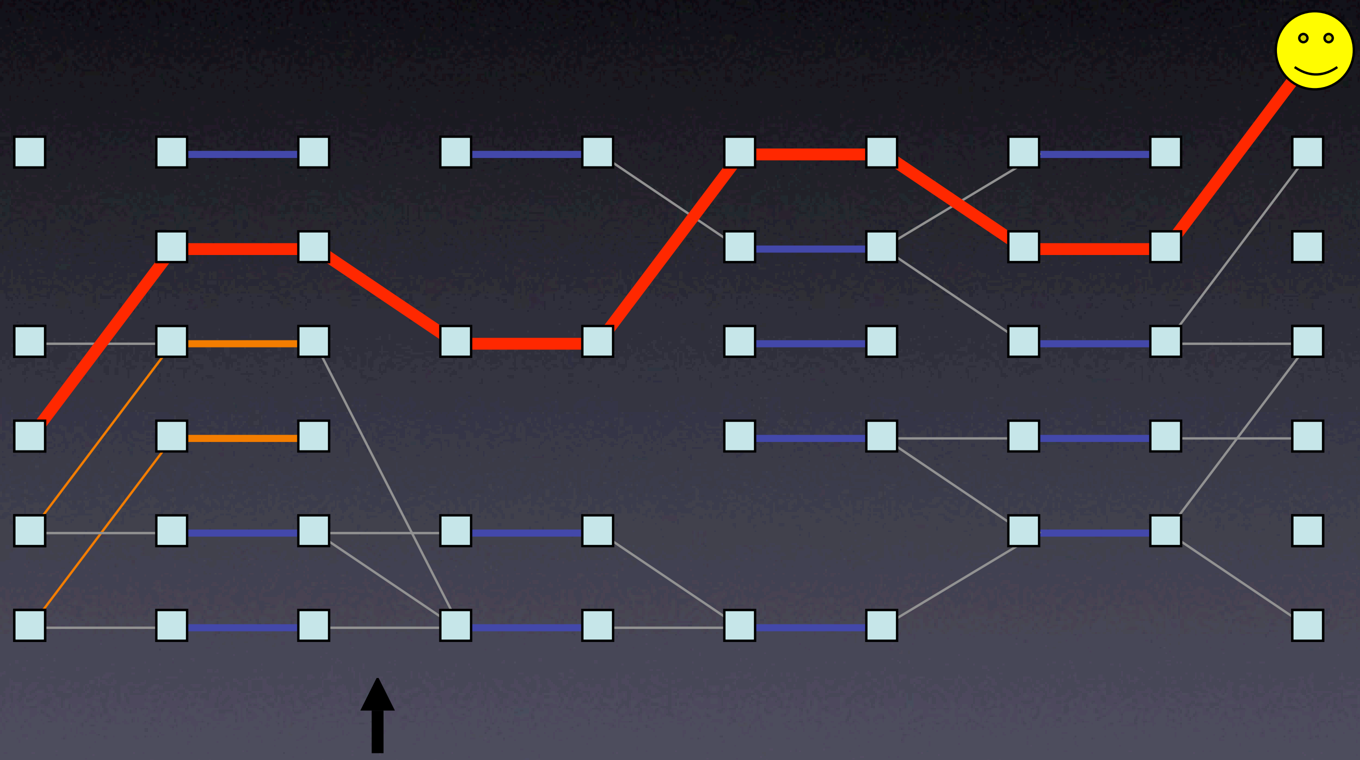


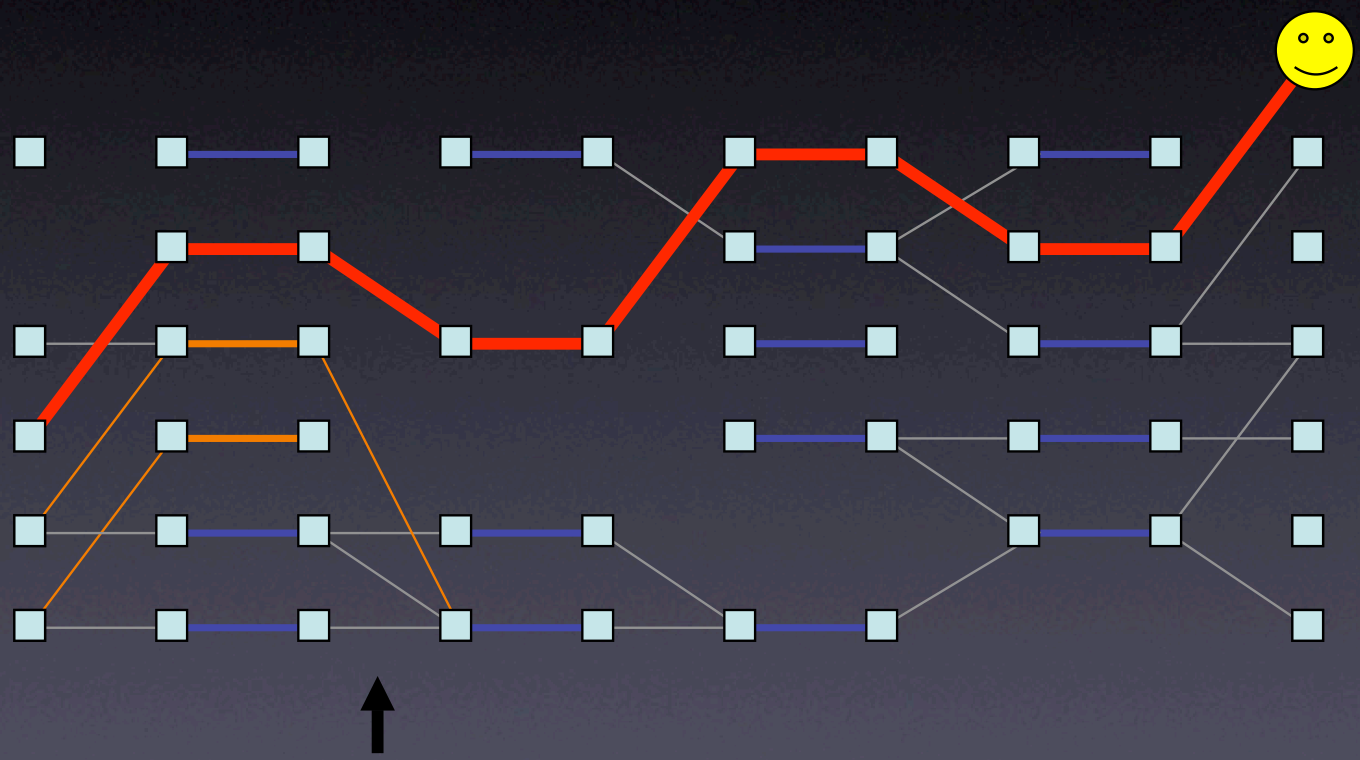


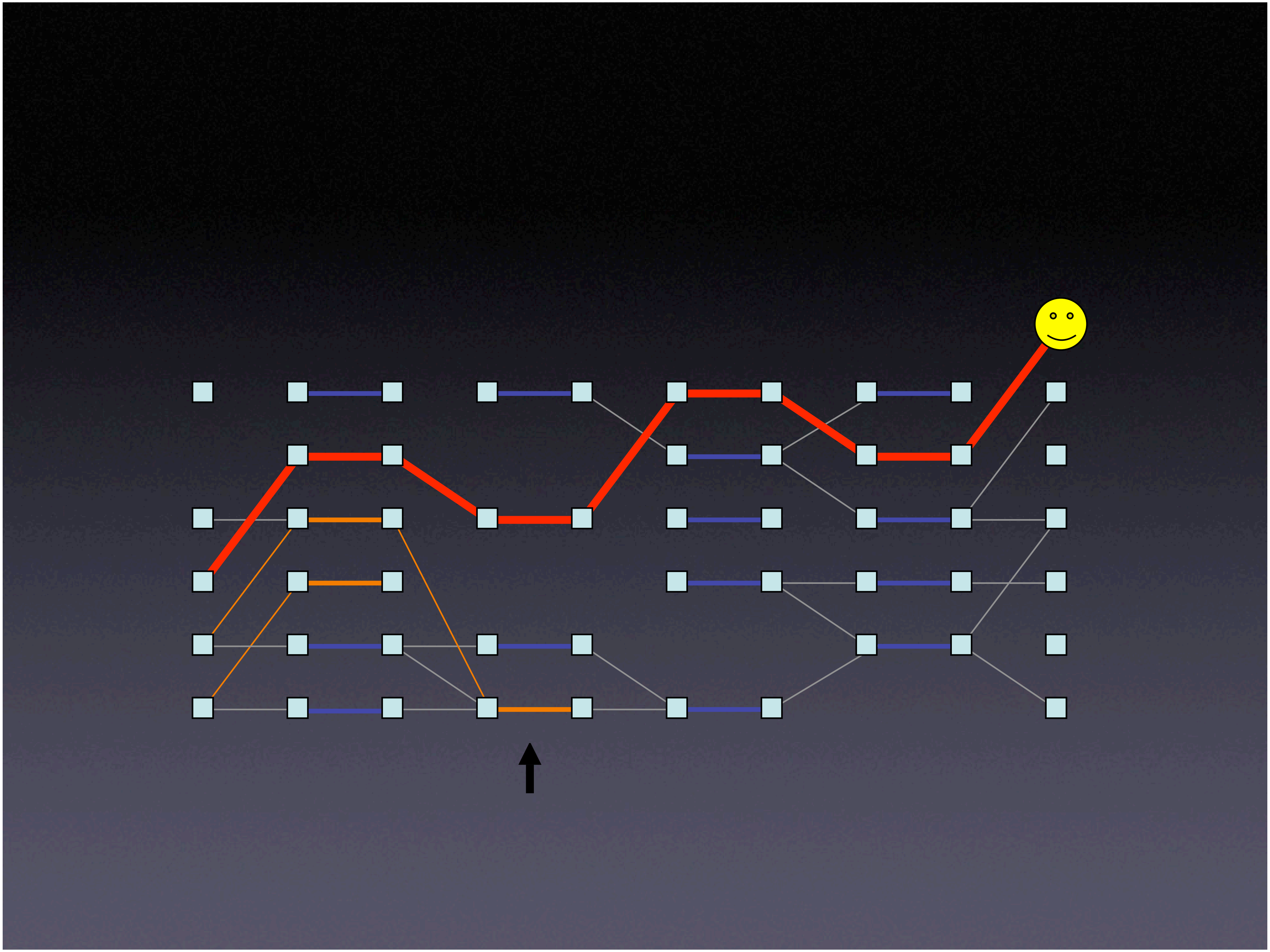


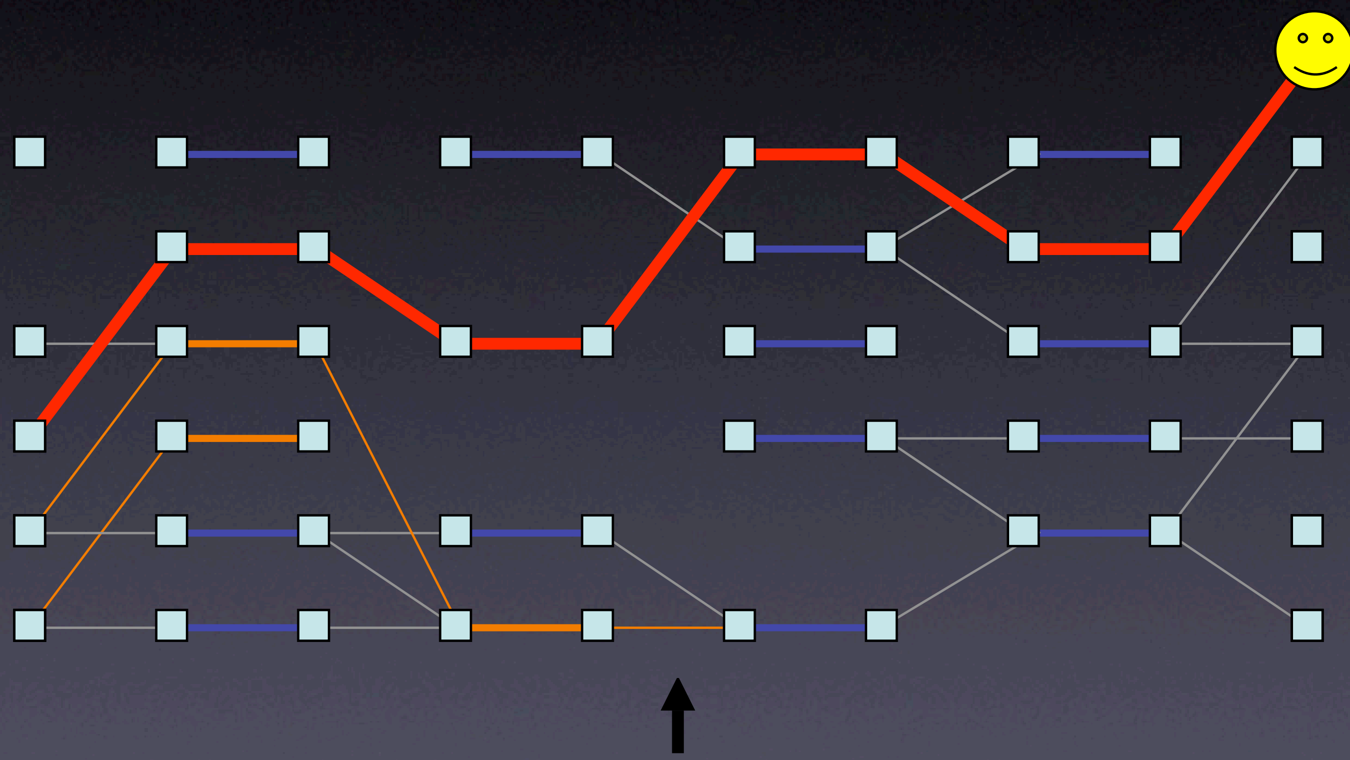


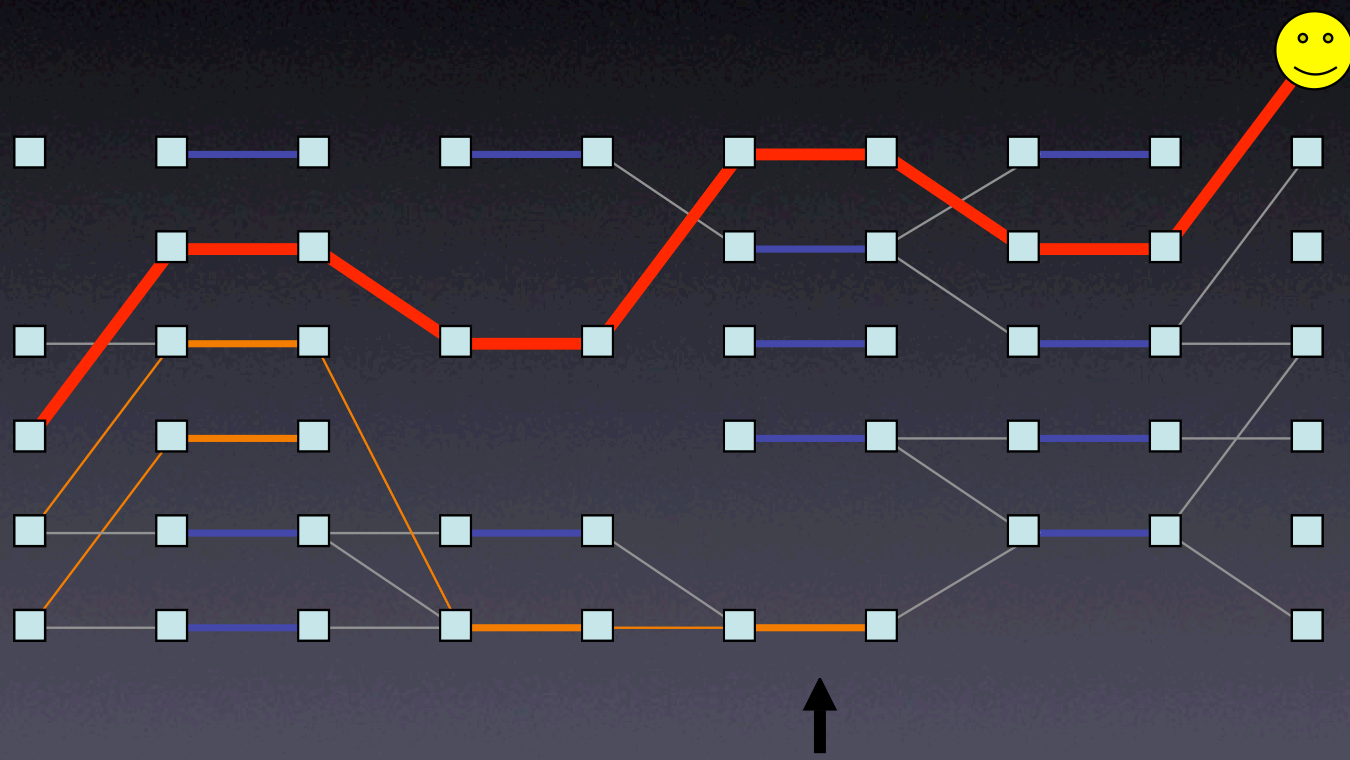


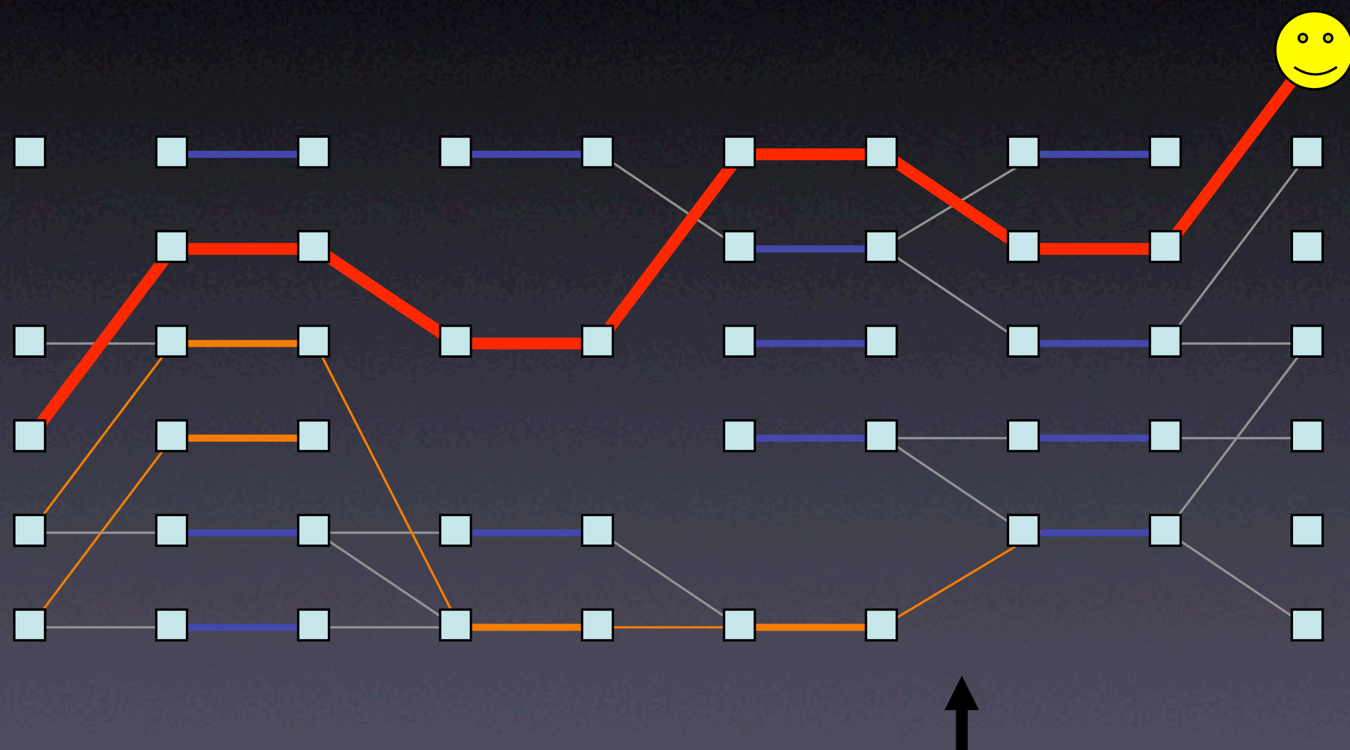


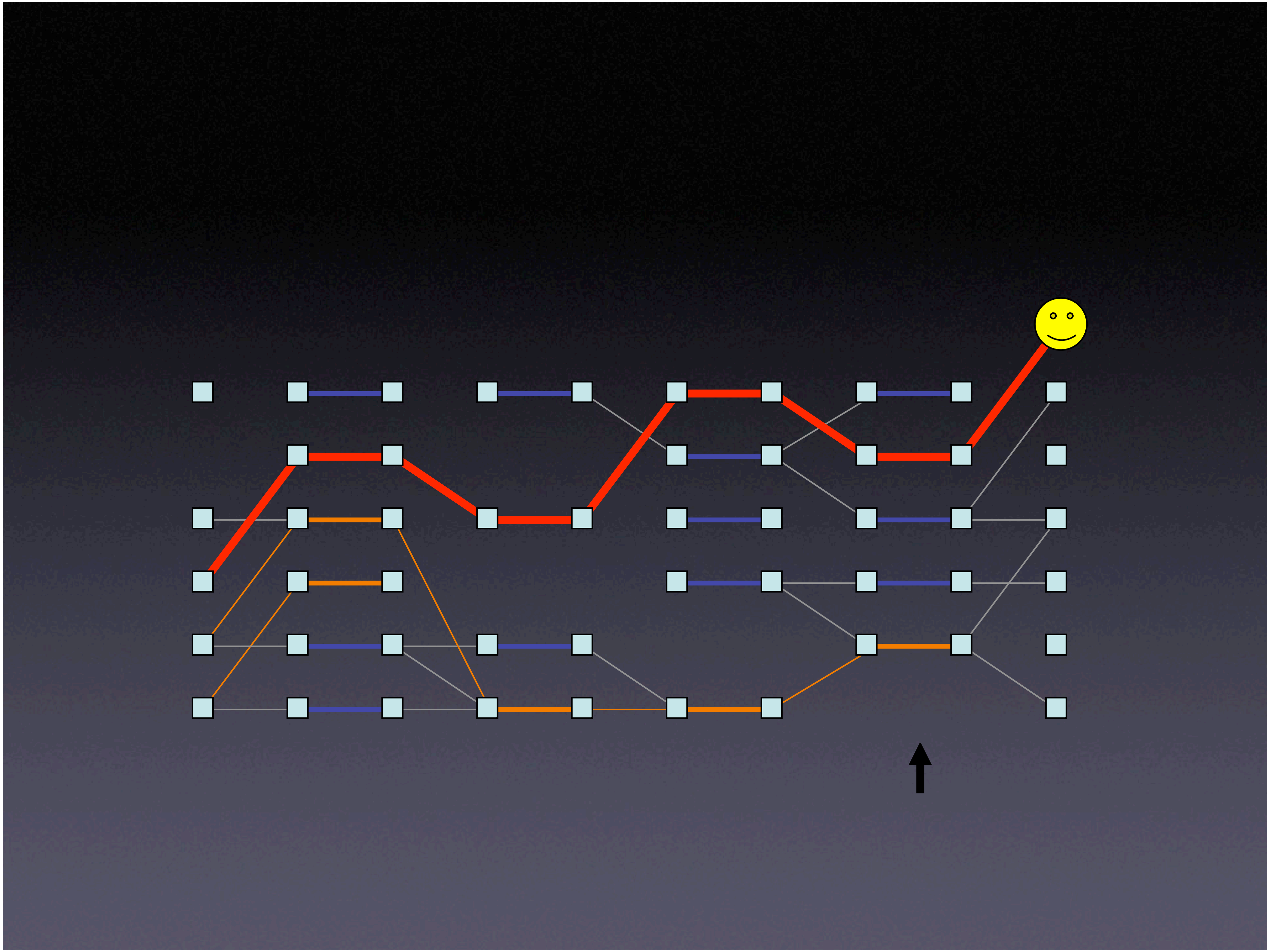


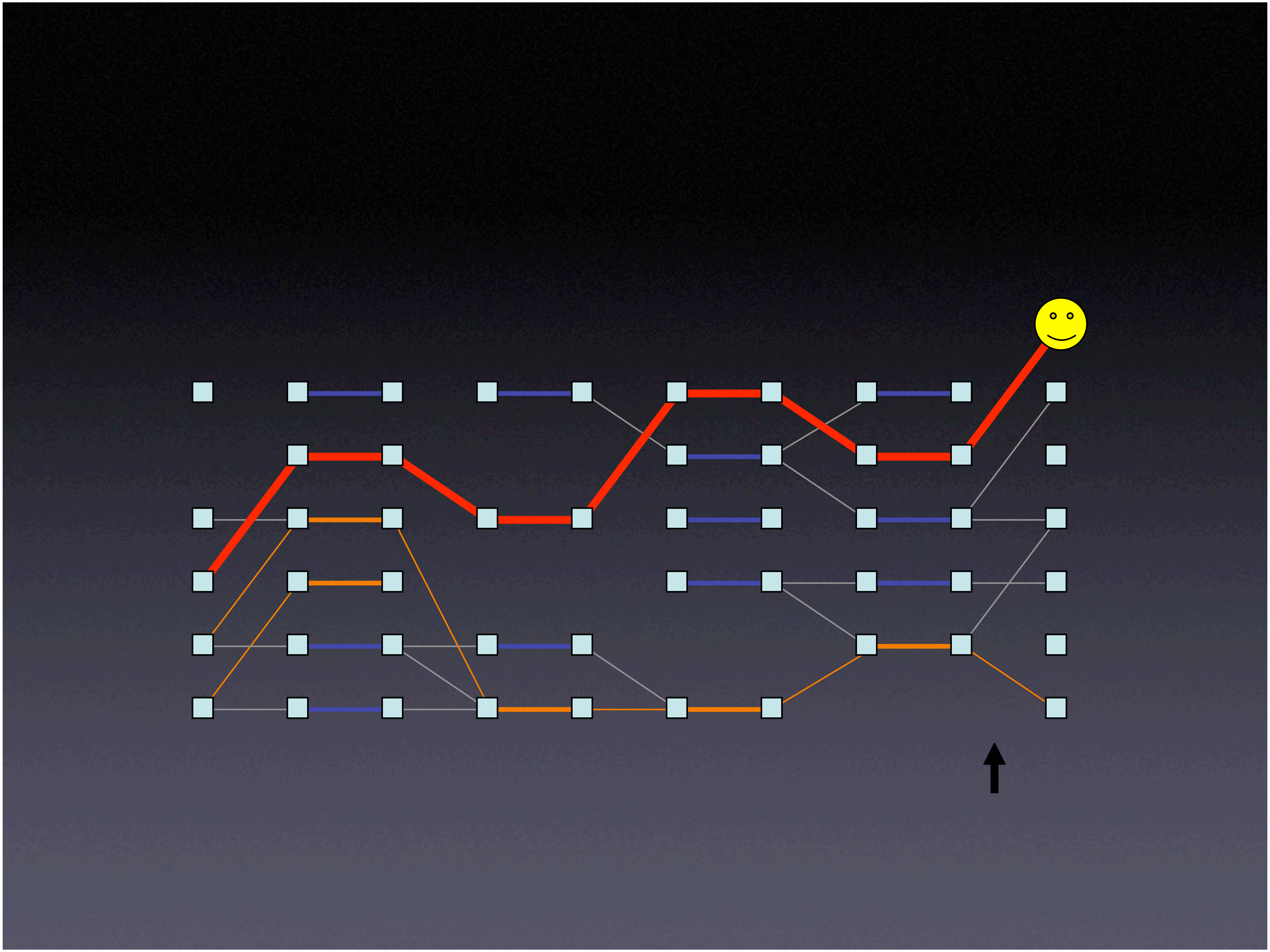


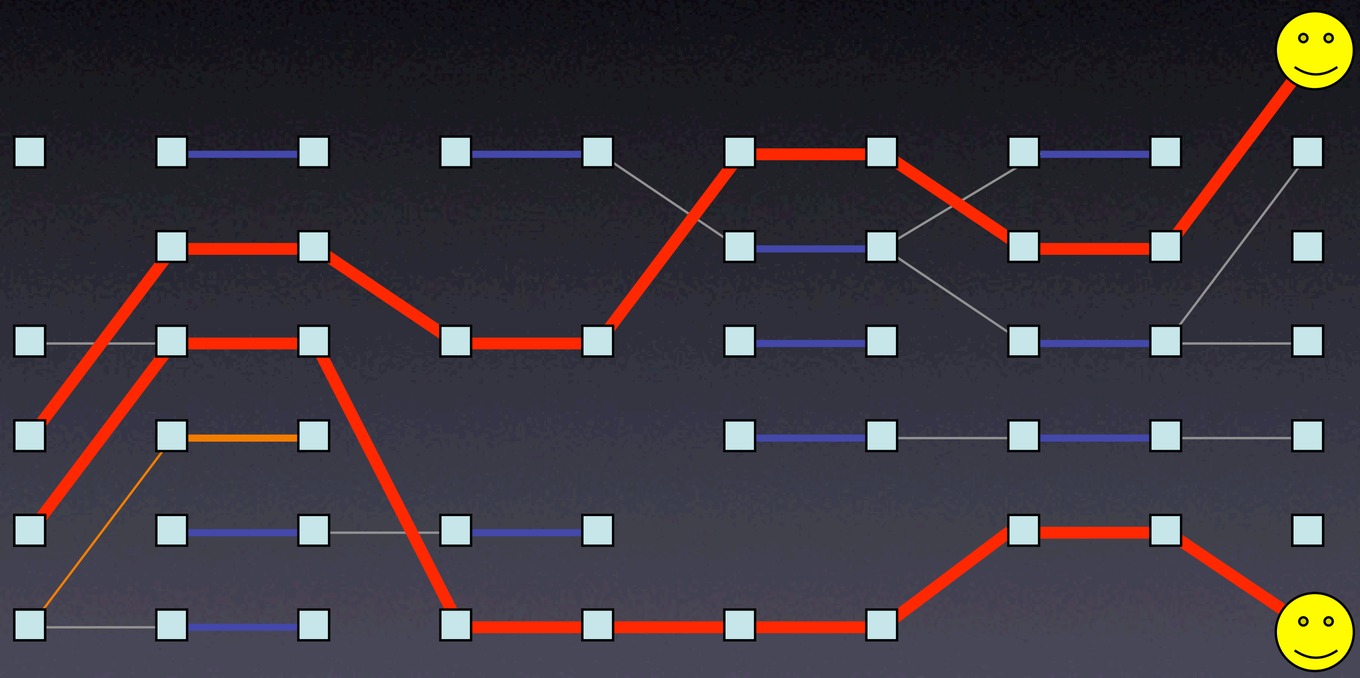




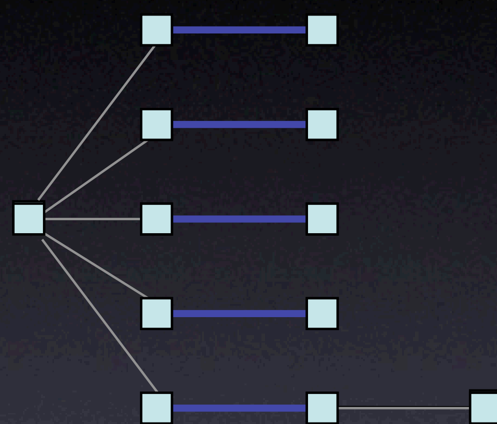








Limiting Backtracking



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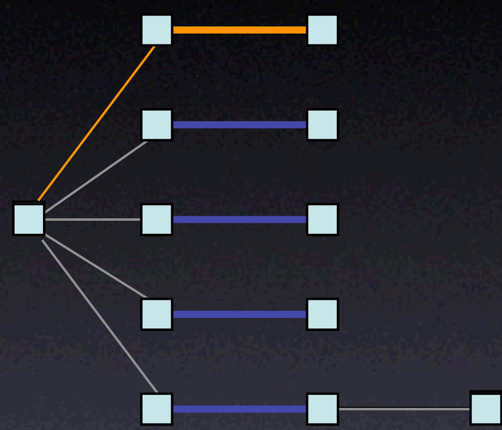

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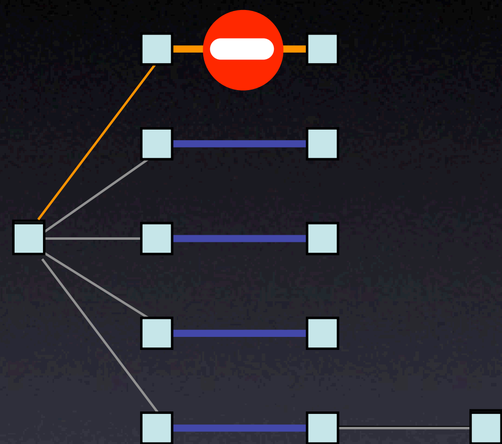

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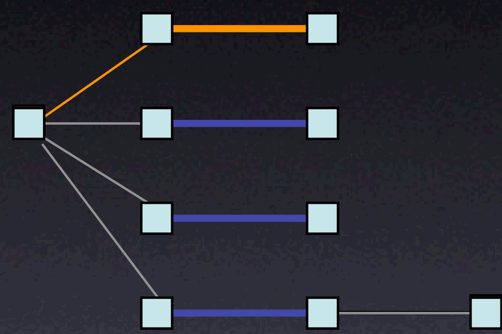

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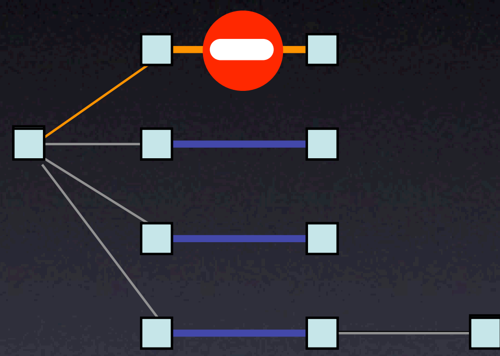

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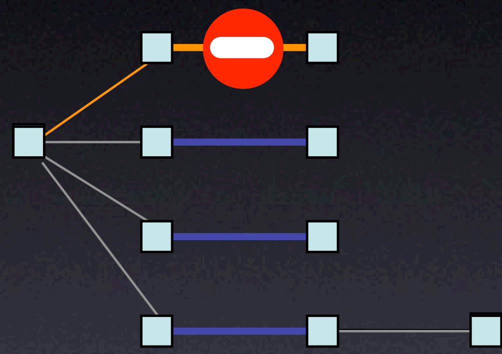

- Solution: If number of paths being grown falls below threshold δn then delete and backtrack.

  Good: Only backtrack a constant number of times

  Bad: Don't find a maximal set of node disjoint paths

- In a constant number of passes, we find a constant fraction of length i node disjoint paths/augmenting paths.

Weighted Matching.

Single Pass $3+2\sqrt{2}$ Approximation

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- Hence $w(OPT) \le (1+\gamma)\, w(T(S)) + 2(1+\gamma)\, w(S) < (3+2\sqrt{2})\, w(S)$

Multi-pass $2+\epsilon$ Approximation

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- Repeat if $|M_i|/ |M_{i-1}|> 1+\kappa(\epsilon)$
- Claim 1: A constant number of passes suffices
- Claim 2: When $|M_i|/ |M_{i-1}| \leq 1+\kappa$ we have a $2+\epsilon$ approx.

Conclusions

- Unweighted Matchings:

  $1+\epsilon$ approximation in constant passes.

- Weighted Matchings:

  $3+2\sqrt{2}$ approximation in single pass.

  $2+\epsilon$ approximation in constant passes.


Thanks.