Optimal Prediction Markets with Optimal Players Learn Optimally for Log Loss

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Prediction Markets Learn Optimally

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Definitions

Player: Someone with an initial endowment of

Optimal Player: A player optimizing expected log wealth after after T rounds.

Prediction Market: A market for securities that pay $1 if an event occurs and $0 otherwise.

Optimal Prediction Market: A Prediction market where the price of the security is such that supply = demand.

Learning Market: A sequece of markets where the security price has small regret in log loss with respect to all players.

(Well known) Optimal Players use Kelly Betting

If w = current wealth, how much should you bet?

$f = 1 \Rightarrow$ lose everything if you are ever wrong

$f = 0 \Rightarrow$ never win anything.

Kelly betting says:

$$f^* = \frac{p - p_m}{1 - p_m}$$

Which is optimal for maximizing expected log wealth.

(Well Known) Log loss regret optimized by Bayes Rule

Suppose you have experts $\{i\}$ which make a prediction p_{it} on round t. How can you compete with the best?

Let w_i = initial "prior" on expert i ($\sum_i w_i = 1$). Bayes rule $\Rightarrow$ weight on expert i is:

$$w_i \prod_{t=1}^{T} \left(\frac{p_{it}}{p_{mt}}\right)^{y_t} \left(\frac{1-p_{it}}{1-p_{mt}}\right)^{1-y_t}$$

where p_{mt} = the wealth weighted average.

Theorem: For all w_i, for all sequences of p_{it} and y:

$$L(\vec{p}_m, \vec{y}) \leq \min_i L(\vec{p}_i, \vec{y}) + \ln \frac{1}{w_i}$$

where $L =$log loss

(new) If every agent bets according to Kelly, wealth is redistributed according to Bayes law.

If $y = 0$, wealth afterwards is $\frac{1-p_i}{1-p_m} w_i$.

if $y = 1$, wealth afterwards is $\frac{p_i}{p_m} w_i$.

Now, connect the dots.

To think about

What happens when the market designer cares about other losses?

What happens when the market player cares about something other losses?

Are market options immoral?