CMPSCI 311: Introduction to Algorithms	**Spring 2018**

Discussion 9

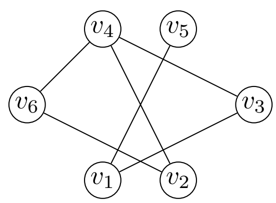
4/20/2018 Name:

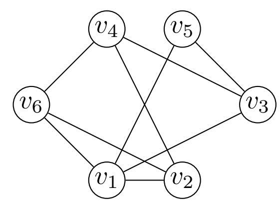
You will be randomly assigned groups to work on these problems in discussion section. List your group members on your worksheet and turn it in at the end of class. Write first and last names. Each group member should turn in their own paper.

1. We say that a graph $G = (V, E)$ is k-colorable, if there exist a *coloring* $\phi : V \to \{1, \dots, k\}$ such that for every $(u, v) \in E$, $\phi(u) \neq \phi(v)$. In words, we can color all of the vertices in the graph, using at most k colors, so that no neighboring vertices have the same color.

   In the graph-coloring problem we are given a graph $G = (V, E)$ and a number k and our goal is decide whether the graph is k-colorable.

   (a) For the following two graphs, what are the smallest values of k such that they are k-colorable.





   (b) In the 2-COLORING problem, our goal is to decide a graph can be colored with 2 colors or not. Design an algorithm for 2-coloring.

   (c) EVEN is the problem of deciding if a given integer x is even. Prove that 2-COLORING $\leq_p$ EVEN.

(d) Prove that 3-COLORING $\leq_P$ INDEPENDENTSET.

(e) We know that INDEPENDENTSET is NP-Complete. Does the above reduction immediately imply that 3-COLORING is also NP-Complete?.