Log-Linear Models
a.k.a. Logistic Regression, Maximum Entropy Models

Introduction to Natural Language Processing
Computer Science 585—Fall 2009
University of Massachusetts Amherst

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(some slides from Jason Eisner and Dan Klein)

Probability is Useful

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  - Bayesian smoothing: max p(θ|data) = max p(θ, data) =p(θ)p(data|θ)

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- **p(…) has to capture our intuitions about the ling. data**

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- Old AI hacking technique:
  - Possible parses (or whatever) have scores.
  - Pick the one with the best score.
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    - Completely ad hoc!
    - Throw anything you want into the stew
    - Add a bonus for this, a penalty for that, etc.

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  - Pos
  - Pic
  - Ho
- “Lear ... uses and pena ... nce. ☺
- Total
  - Ca ... e

Exposé at 9

Probabilistic Revolution Not Really a Revolution, Critics Say

Log-probabilities no more than scores in disguise

“We’re just adding stuff up like the old corrupt regime did,” admits spokesperson

Nuthin' but adding weights

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- *Note:* Today we'll use +logprob not –logprob:
  i.e., bigger weights are **better**.

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- PCFG: log p(NP VP | S) + log p(Papa | NP) + log p(VP PP | VP) …
  - Can regard any linguistic object as a collection of features (here, tree = a collection of context-free rules)
  - Weight of the object = total weight of features
  - Our weights have always been conditional log-probs (≤ 0)
    - but that is going to change in a few minutes!
- HMM tagging: … + log p(t7 | t5, t6) + log p(w7 | t7) + …
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   - Multiply indep. conditional probs – normalized, unlike scores
   - p(English text) * p(English phonemes | English text) * p(Jap. phonemes | English phonemes) * p(Jap. text | Jap. phonemes)
   - p(semantics) * p(syntax | semantics) * p(morphology | syntax) * p(phonology | morphology) * p(sounds | phonology)

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- Consider e.g. Naïve Bayes for text categorization:
  - `Buy this supercalifragilistic Ginsu knife set for only $39 today …`
- Some useful features:
  - Contains `Buy`
  - Contains `supercalifragilistic`
  - Contains a dollar amount under `$100`
  - Contains an imperative sentence
  - Reading level = 8th grade
  - Mentions money (use word classes and/or regexp to detect this)

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spam | ling

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Naïve Bayes claims .5*.9=45% of spam has **both** features – 25*9=225x more likely than in ling.

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*but **here** are the emails with both features – only 25x!*

.9 .1

90% of spam has this – 9x more likely than in ling

  - Mentions money

Naïve Bayes claims .5*.9=45% of spam has **both** features – 25*9=225x more likely than in ling.

- Naïve Bayes: pick C maximizing p(C) * p(feat 1 | C) * …
- What assumption does Naïve Bayes make? True here?

Probabilists Regret Being Bound by Principle

- But ad-hoc approach does have one advantage
  - **Can adjust scores to compensate for feature overlap …**
- Some useful features of this message:

spam | ling

.5 | .02
  - Contains a dollar amount under `$100`

.9 | .1
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spam	ling		spam	ling
.5	.02	Contains a dollar amount under $100	-1	-5.6
.9	.1	Mentions money	-.15	-3.3

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	spam	ling	log prob spam	log prob ling	adjusted spam	adjusted ling
Contains a dollar amount under $100	.5	.02	-1	-5.6	-.85	-2.3
Mentions money	.9	.1	-.15	-3.3	-.15	-3.3

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Revolution Corrupted by Bourgeois Values

12

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- Naïve Bayes needs overlapping but independent features

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- Naïve Bayes needs overlapping but independent features
- But not clear how to restructure these features like that:
  - Contains `Buy`
  - Contains `supercalifragilistic`
  - Contains a dollar amount under `$100`
  - Contains an imperative sentence
  - Reading level = 7th grade
  - Mentions money (use word classes and/or regexp to detect this)
  - …

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  - +4 Contains `Buy`
  - +0.2 Contains `supercalifragilistic`
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  - -3 Reading level = 7th grade
  - +5 Mentions money (use word classes and/or regexp to detect this)
  - … …

total: 5.77

- Boy, we'd like to be able to throw all that useful stuff in without worrying about feature overlap/independence.
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- Well, maybe we can add up scores and pretend like we got a log probability: **log p(feats | spam) = 5.77**
- Oops, then p(feats | spam) = exp 5.77 = 320.5

Renormalize by 1/Z to get a

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- $p(\text{feats} \mid \text{spam}) = \exp 5.77 = 320.5$
- $p(m \mid \text{spam}) = (1/Z(\lambda)) \exp \sum_i \lambda_i f_i(m)$ where

  m is the email message

  λ_i is weight of feature i

  $f_i(m) \in \{0,1\}$ according to whether m has feature i

  More generally, allow $f_i(m)$ = count or strength of feature.

  $1/Z(\lambda)$ is a normalizing factor making $\sum_m p(m \mid \text{spam})=1$

  (summed over all possible messages m! hard to find!)

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- The weights we add up are basically arbitrary.
- They don't have to mean anything, so long as they give us a good probability.
- Why is it called "log-linear"?

Why Bother?

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  - Choose weights λ_i that maximize logprob of labeled training data = $\log \prod_j p(c_j)\, p(m_j \mid c_j)$
    - where $c_j \in \{\text{ling}, \text{spam}\}$ is classification of message m_j
    - and $p(m_j \mid c_j)$ is log-linear model from previous slide
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  - Convex function – easy to maximize! (why?)
- **But:** $p(m_j \mid c_j)$ for a given λ requires $Z(\lambda)$: hard!

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Attempt to Cancel out Z

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- Set weights to maximize $\prod_j p(c_j)\, p(m_j \mid c_j)$
  - where $p(m \mid \text{spam}) = (1/Z(\lambda)) \exp \sum_i \lambda_i f_i(m)$
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- But we can fix this …

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  p(m|spam)*p(spam) vs. p(m|ling)*p(ling)

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  gives us both p(m,spam) and p(m,ling)
- Equivalent to changing feature set to:
  - spam
  - spam and Contains `Buy`
  - spam and Contains `supercalifragilistic`
  - …
  - ling
  - ling and Contains `Buy`
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- Equivalent to changing feature set to:
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  - spam and Contains `Buy` ←old spam model's weight for "contains Buy"
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  - ling
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  - spam ← weight of this feature is log p(spam) + a constant
  - spam and Contains `Buy` ←old spam model's weight for "contains Buy"
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  - …
  - ling ← weight of this feature is log p(ling) + a constant
  - ling and Contains `Buy` ←old ling model's weight for "contains Buy"
  - ling and Contains `supercalifragilistic`
- No real change, but 2 categories now share single feature set and single value of Z(λ)

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Now $p(m,c) = (1/Z(\lambda)) \exp \sum_i \lambda_i f_i(m,c)$ where $c \in \{\text{ling, spam}\}$

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  - p(spam | m) = p(m,spam) / (p(m,spam) + p(m,ling))

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    $= \exp \sum_i \lambda_i f_i(m,spam) / (\exp \sum_i \lambda_i f_i(m,spam) + \exp \sum_i \lambda_i f_i(m,ling))$

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- Now Z cancels out of conditional probability!
  - $p(\text{spam} \mid m) = p(m,\text{spam}) / (p(m,\text{spam}) + p(m,\text{ling}))$

    $= \exp \sum_i \lambda_i f_i(m,\text{spam}) / (\exp \sum_i \lambda_i f_i(m,\text{spam}) + \exp \sum_i \lambda_i f_i(m,\text{ling}))$
  - Easy to compute now …
  - $\prod_j p(c_j \mid m_j)$ is still convex, so easy to maximize too

Generative vs. Conditional

- What is the most likely label for a given input?
- How likely is a given label for a given input?
- What is the most likely input value?
- How likely is a given input value?
- How likely is a given input value with a given label?
- What is the most likely label for an input that might have one of two values (but we don't know which)?

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- Suppose I tell you that 55% of all messages are in class A.
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- Suppose I also tell you that 10% of all messages contain `Buy` and 80% of these are in class A or C.

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- Suppose I also tell you that 10% of all messages contain `Buy` and 80% of these are in class A or C.
- Question: Now what is your guess for p(C | m), if m contains `Buy`?

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- Suppose there are 10 classes, A through J.
- I don't give you any other information.
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- Suppose I tell you that 55% of all messages are in class A.
- Question: Now what is your guess for p(C | m)?

- Suppose I also tell you that 10% of all messages contain `Buy` and 80% of these are in class A or C.
- Question: Now what is your guess for p(C | m), if m contains `Buy`?
- OUCH!

Maximum Entropy

	A	B	C	D	E	F	G	H	I	J
Buy	0.051	0.003	0.029	0.003	0.003	0.003	0.003	0.003	0.003	0.003
Other	0.499	0.045	0.045	0.045	0.045	0.045	0.045	0.045	0.045	0.045

- Column A sums to 0.55 ("55% of all messages are in class A")

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Maximum Entropy

	A	B	C	D	E	F	G	H	I	J
Buy	0.051	0.003	0.029	0.003	0.003	0.003	0.003	0.003	0.003	0.003
Other	0.499	0.045	0.045	0.045	0.045	0.045	0.045	0.045	0.045	0.045

- Column A sums to 0.55
- Row `Buy` sums to 0.1 ("10% of all messages contain `Buy`")

Maximum Entropy

	A	B	C	D	E	F	G	H	I	J
Buy	0.051	0.003	0.029	0.003	0.003	0.003	0.003	0.003	0.003	0.003
Other	0.499	0.045	0.045	0.045	0.045	0.045	0.045	0.045	0.045	0.045

- Column A sums to 0.55
- Row `Buy` sums to 0.1
- (`Buy`, A) and (`Buy`, C) cells sum to 0.08 ("80% of the 10%")

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Maximum Entropy

	A	B	C	D	E	F	G	H	I	J
Buy	0.051	0.003	0.029	0.003	0.003	0.003	0.003	0.003	0.003	0.003
Other	0.499	0.045	0.045	0.045	0.045	0.045	0.045	0.045	0.045	0.045

- Column A sums to 0.55
- Row `Buy` sums to 0.1
- (`Buy`, A) and (`Buy`, C) cells sum to 0.08 ("80% of the 10%")
- Given these constraints, fill in cells "as equally as possible": maximize the entropy (related to cross-entropy, perplexity)

Maximum Entropy

	A	B	C	D	E	F	G	H	I	J
Buy	0.051	0.003	0.029	0.003	0.003	0.003	0.003	0.003	0.003	0.003
Other	0.499	0.045	0.045	0.045	0.045	0.045	0.045	0.045	0.045	0.045

- Column A sums to 0.55
- Row `Buy` sums to 0.1
- (`Buy`, A) and (`Buy`, C) cells sum to 0.08 ("80% of the 10%")
- Given these constraints, fill in cells "as equally as possible": maximize the entropy (related to cross-entropy, perplexity)

Entropy = -.051 log .051 - .0025 log .0025 - .029 log .029 - …

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Largest if probabilities are evenly distributed

Maximum Entropy

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- (`Buy`, A) and (`Buy`, C) cells sum to 0.08 ("80% of the 10%")
- Given these constraints, fill in cells "as equally as possible": maximize the entropy
- Now p(`Buy`, C) = .029 and p(C | `Buy`) = .29
- We got a compromise: p(C | `Buy`) < p(A | `Buy`) < .55

Generalizing to More Features

<$100

Other

	A	B	C	D	E	F	G	H	…
Buy	0.051	0.003	0.029	0.003	0.003	0.003	0.003	0.003	
Other	0.499	0.045	0.045	0.045	0.045	0.045	0.045	0.045	

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  $p(m,c) = (1/Z(\lambda)) \exp \sum_i \lambda_i f_i(m,c)$

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- Gives another motivation for our log-linear approach.

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Log-linear form derivation

- Say we are given some ***constraints*** in the form of feature expectations:

$$\sum_x p(x) f_i(x) = \alpha_i$$

- In general, there may be many distributions p(x) that satisfy the constraints. Which one to pick?
- The one with maximum entropy (making fewest possible additional assumptions---Occum's Razor)
- This yields an optimization problem

$$\max H(p(x)) = -\sum_x p(x) \log p(x)$$

$$\text{Subject to } \sum_x p(x) f_i(x) = \alpha_i, \forall i \text{ and } \sum_x p(x) = 1$$

Log-linear form derivation

- To solve the maxent problem, we use Lagrange multipliers:

$$L = -\sum_{\mathbf{x}} p(\mathbf{x}) \log p(\mathbf{x}) - \sum_i \theta_i \left(\sum_{\mathbf{x}} p(\mathbf{x}) f_i(\mathbf{x}) - \alpha_i \right) - \mu \left(\sum_{\mathbf{x}} p(\mathbf{x}) - 1 \right)$$

$$\frac{\partial L}{\partial p(\mathbf{x})} = 1 + \log p(\mathbf{x}) - \sum_i \theta_i f_i(\mathbf{x}) - \mu$$

$$p^*(\mathbf{x}) = e^{\mu - 1} \exp \left\{ \sum_i \theta_i f_i(\mathbf{x}) \right\}$$

$$Z(\theta) = e^{1-\mu} = \sum_{\mathbf{x}} \exp \left\{ \sum_i \theta_i f_i(\mathbf{x}) \right\}$$

$$p(\mathbf{x}|\theta) = \frac{1}{Z(\theta)} \exp \left\{ \sum_i \theta_i f_i(\mathbf{x}) \right\}$$

- So feature constraints + maxent implies exponential family.
- Problem is convex, so solution is unique.

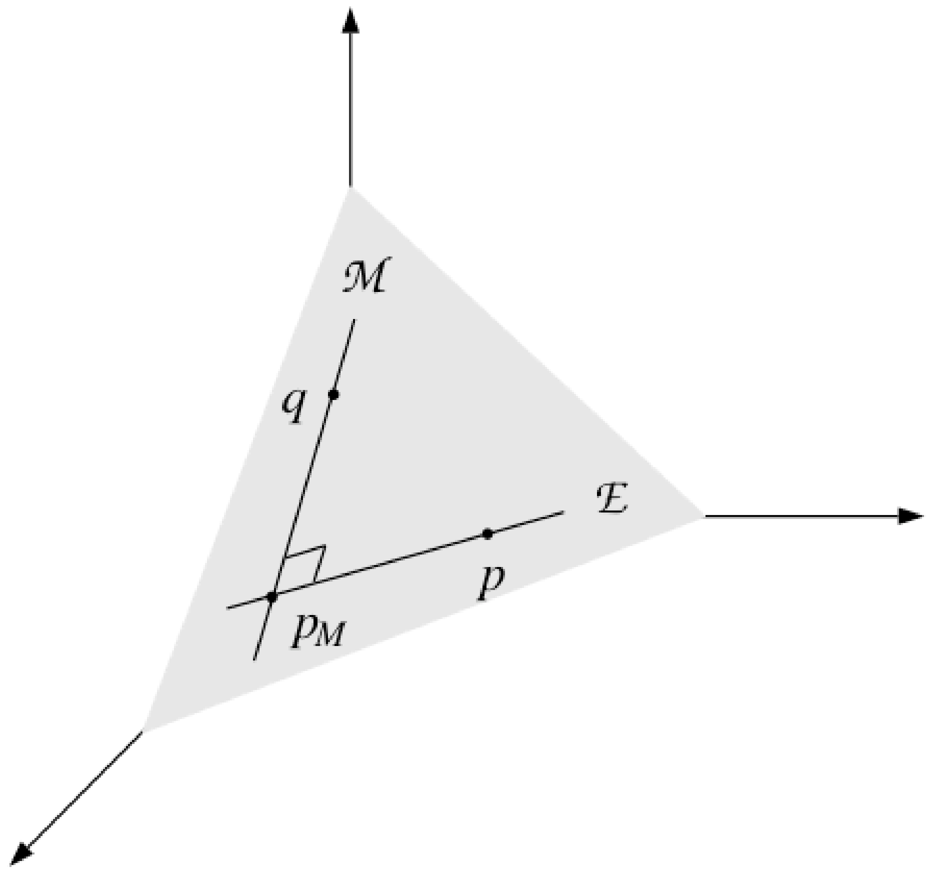
MaxEnt = Max Likelihood

Define two submanifolds on the probability simplex $p(\mathbf{x})$.

The first is $\mathcal{E}$, the set of all exponential family distributions based on a particular set of features $f_i(\mathbf{x})$.

The second is $\mathcal{M}$, the set of all distributions that satisfy the feature expectation constraints.

They intersect at a single distribution p_M, the maxent, maximum likelihood



Exponential Model Likelihood

- Maximum Likelihood (Conditional) Models :
  - Given a model form, choose values of parameters to maximize the (conditional) likelihood of the data.

- Exponential model form, for a data set (C,D):

$$\log P(C \mid D, \lambda) = \sum_{(c,d)\in(C,D)} \log P(c \mid d, \lambda) = \sum_{(c,d)\in(C,D)} \log \frac{\exp \sum_i \lambda_i f_i(c,d)}{\sum_{c'} \exp \sum_i \lambda_i f_i(c',d)}$$

Building a Maxent Model

- Define features (indicator functions) over data points.
  - Features represent sets of data points which are distinctive enough to deserve model parameters.
  - Usually features are added incrementally to "target" errors.

- For any given feature weights, we want to be able to calculate:
  - Data (conditional) likelihood
  - Derivative of the likelihood wrt each feature weight
    - Use expectations of each feature according to the model

- Find the optimum feature weights (next part).

The Likelihood Value

- The (log) conditional likelihood is a function of the iid data (C,D) and the parameters λ:

$$\log P(C \mid D, \lambda) = \log \prod_{(c,d)\in(C,D)} P(c \mid d, \lambda) = \sum_{(c,d)\in(C,D)} \log P(c \mid d, \lambda)$$

- If there aren't many values of *c*, it's easy to calculate:

$$\log P(C \mid D, \lambda) = \sum_{(c,d)\in(C,D)} \log \frac{\exp \sum_i \lambda_i f_i(c,d)}{\sum_{c'} \exp \sum_i \lambda_i f_i(c,d)}$$

- We can separate this into two components:

$$\log P(C \mid D, \lambda) = \sum_{(c,d)\in(C,D)} \log \exp \sum_i \lambda_i f_i(c,d) - \sum_{(c,d)\in(C,D)} \log \sum_{c'} \exp \sum_i \lambda_i f_i(c',d)$$

$$\log P(C \mid D, \lambda) = N(\lambda) - M(\lambda)$$

- The derivative is the difference between the derivatives of each component

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The Derivative I: Numerator

$$\frac{\partial N(\lambda)}{\partial \lambda_i} = \frac{\partial \sum_{(c,d)\in(C,D)} \log \exp \sum_i \lambda_{ci} f_i(c,d)}{\partial \lambda_i} = \frac{\partial \sum_{(c,d)\in(C,D)} \sum_i \lambda_i f_i(c,d)}{\partial \lambda_i}$$

$$= \sum_{(c,d)\in(C,D)} \frac{\partial \sum_i \lambda_i f_i(c,d)}{\partial \lambda_i}$$

$$= \sum_{(c,d)\in(C,D)} f_i(c,d)$$

Derivative of the numerator is: the empirical count(f_i, c)

The Derivative II: Denominator

$$\frac{\partial M(\lambda)}{\partial \lambda_i} = \frac{\partial \sum_{(c,d)\in(C,D)} \log \sum_{c'} \exp \sum_i \lambda_i f_i(c',d)}{\partial \lambda_i}$$

$$= \sum_{(c,d)\in(C,D)} \frac{1}{\sum_{c''} \exp \sum_i \lambda_i f_i(c'',d)} \frac{\partial \sum_{c'} \exp \sum_i \lambda_i f_i(c',d)}{\partial \lambda_i}$$

$$= \sum_{(c,d)\in(C,D)} \frac{1}{\sum_{c''} \exp \sum_i \lambda_i f_i(c'',d)} \sum_{c'} \frac{\exp \sum_i \lambda_i f_i(c',d)}{1} \frac{\partial \sum_i \lambda_i f_i(c',d)}{\partial \lambda_i}$$

$$= \sum_{(c,d)\in(C,D)} \sum_{c'} \frac{\exp \sum_i \lambda_i f_i(c',d)}{\sum_{c''} \exp \sum_i \lambda_i f_i(c'',d)} \frac{\partial \sum_i \lambda_i f_i(c',d)}{\partial \lambda_i}$$

$$= \sum_{(c,d)\in(C,D)} \sum_{c'} P(c'|d,\lambda) f_i(c',d)$$ = predicted count(f_i, λ)

The Derivative III

$$\frac{\partial \log P(C \mid D, \lambda)}{\partial \lambda_i} = \text{actual count}(f_i, C) - \text{predicted count}(f_i, \lambda)$$

- The optimum parameters are the ones for which each feature's predicted expectation equals its empirical expectation. The optimum distribution is:
  - Always unique (but parameters may not be unique)
  - Always exists (if features counts are from actual data).

- Features can have high model expectations (predicted counts) either because they have large weights or because they occur with other features which have large weights.

Summary

- We have a function to optimize:

$$\log P(C \mid D, \lambda) = \sum_{(c,d)\in(C,D)} \log \frac{\exp \sum_i \lambda_i f_i(c,d)}{\sum_{c'} \exp \sum_i \lambda_i f_i(c,d)}$$

- We know the function's derivatives:

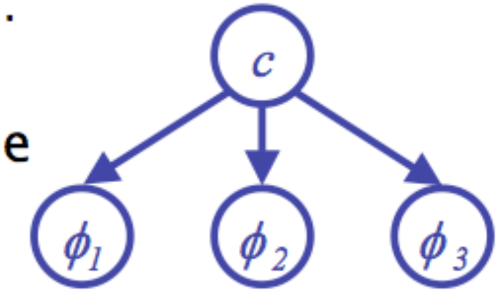
$$\partial \log P(C \mid D, \lambda) / \partial \lambda_i = \text{actual count}(f_i, C) - \text{predicted count}(f_i, \lambda)$$

- Perfect situation for general optimization (Part II)

By gradient ascent or conjugate gradient.

Comparison to Naïve-Bayes

- Naïve-Bayes is another tool for classification:
  - We have a bunch of random variables (data features) which we would like to use to predict another variable (the class):



  - The Naïve-Bayes likelihood over classes is:

$$P(c \mid d, \lambda) = \frac{P(c)\prod_i P(\phi_i \mid c)}{\sum_{c'} P(c')\prod_i P(\phi_i \mid c')} \Rightarrow \frac{\exp\left[\log P(c) + \sum_i \log P(\phi_i \mid c)\right]}{\sum_{c'} \exp\left[\log P(c') + \sum_i \log P(\phi_i \mid c')\right]}$$

$$\Rightarrow \frac{\exp\left[\sum_i \lambda_{ic} f_{ic}(d,c)\right]}{\sum_{c'} \exp\left[\sum_i \lambda_{ic'} f_{ic'}(d,c')\right]}$$

Naïve-Bayes is just an exponential model.

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Comparison to Naïve-Bayes

- The primary differences between Naïve-Bayes and maxent models are:

Naïve-Bayes	Maxent
Trained to maximize joint likelihood of data and classes.	Trained to maximize the conditional likelihood of classes.
Features assumed to supply independent evidence.	Features weights take feature dependence into account.
Feature weights can be set independently.	Feature weights must be mutually estimated.
Features must be of the conjunctive $\Phi(d) \wedge c = c_i$ form.	Features need not be of the conjunctive form (but usually are).

Overfitting

- If we have too many features, we can choose weights to model the training data perfectly.
- If we have a feature that only appears in spam training, not ling training, it will get weight ∞ to maximize p(spam | feature) at 1.
- These behaviors overfit the training data.
- Will probably do poorly on test data.

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Solutions to Overfitting

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   - Require every feature to occur > 4 times, and > 0 times with ling, and > 0 times with spam.

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   - Add one at a time, always greedily picking the one that most improves performance on held-out data.

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2. Only keep 1000 features.
   - Add one at a time, always greedily picking the one that most improves performance on held-out data.
3. Smooth the observed feature counts.
4. Smooth the weights by using a prior.
   - max $p(\lambda|\text{data})$ = max $p(\lambda, \text{data}) = p(\lambda)p(\text{data}|\lambda)$
   - decree $p(\lambda)$ to be high when most weights close to 0

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Smoothing: Priors (MAP)

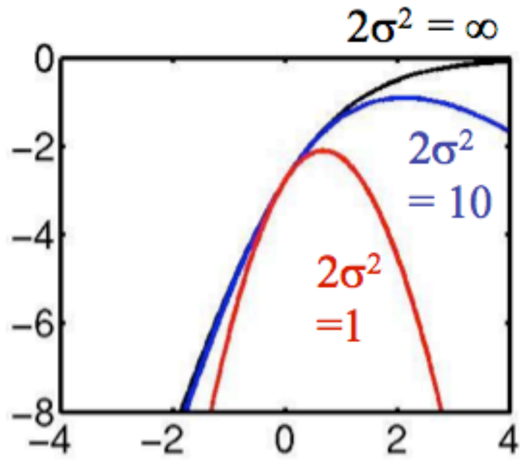
- What if we had a prior expectation that parameter values wouldn't be very large?
- We could then balance evidence suggesting large parameters (or infinite) against our prior.
- The evidence would never totally defeat the prior, and parameters would be smoothed (and kept finite!).
- We can do this explicitly by changing the optimization objective to maximum posterior likelihood:

$$\log P(C,\lambda \mid D) = \log P(\lambda) + \log P(C \mid D,\lambda)$$

Posterior Prior Evidence

Smoothing: Priors

- Gaussian, or quadratic, priors:
  - Intuition: parameters shouldn't be large.
  - Formalization: prior expectation that each parameter will be distributed according to a gaussian with mean μ and variance σ^2.



$$P(\lambda_i) = \frac{1}{\sigma_i \sqrt{2\pi}} \exp\left(-\frac{(\lambda_i - \mu_i)^2}{2\sigma_i^2} \right)$$

  - Penalizes parameters for drifting to far from their mean prior value (usually μ=0).
  - $2\sigma^2$=1 works surprisingly well.

Recipe for a Conditional MaxEnt Classifier

1. Gather *constraints* from training data:

$$\alpha_{iy} = \tilde{E}[f_{iy}] = \sum_{x_j, y_j \in D} f_{iy}(x_j, y_j)$$

2. Initialize all parameters to zero.
3. Classify training data with current parameters. Calculate *expectations*.

$$E_\Theta[f_{iy}] = \sum_{x_j \in D} \sum_{y'} p_\Theta(y'|x_j) f_{iy}(x_j, y')$$

4. Gradient is $\tilde{E}[f_{iy}] - E_\Theta[f_{iy}]$
5. Take a step in the direction of the gradient
6. Until convergence, return to step 3.