CS370
Pinhole camera (HW5) solution

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1 Pinhole camera with CCDs at different distances

There are two points to note in this question.
(A) As the distance from the pinhole to the CCD increases, the amount of light falling per unit area of the CCD will decrease (as a inverse square function of the distance).
(B) Since the red CCD is nearer to the pinhole, the full image seen in the blue and green CCDs will appear in a smaller region in the center of the red CCD. To reconstruct the color image from the three CCDs, we need to extract the pixels in this region of the red CCD and enlarge the resulting sub-image to make it the same size as the images from the other CCDs.

Lets assume that each individual pixel in all three CCDs will have the same physical area. Although one could have a solution where the pixels in the red CCD have a different area compared to the pixels in the blue and green CCDs, this is not the intent in this HW. We also assume that the three CCDs all have the same number of pixels. However, note that this means that the image that appears on the green and blue CCDs will appear only on a smaller number of pixels in the center of the red CCD. To put together the three colors to reconstruct the image, we would take these center pixels in the red CCD and expand the sub-image formed by these pixels to be the same number of pixels as the other CCDs.

(A)Lets assume that all distances are $d1 = 1$, $d2 = 2$, $d3 = 1$ unit. This makes the red CCD have distance of 2 units from pinhole and other CCDs have distance of 4 units from the pinhole. This assumption is not essential, and is only made so that the problem becomes easier to solve. The actual distance values assumed can also be differnent from the ones above.

Note that absorption is essentially wasted light. Since we want to use as much light as possible, we will design a solution with minimum absorption in both images ($m_{1a} = m_{2a} = 0.1$).

Secondly, since we want the same amount of light falling on both the green and blue CCDs, m2 must have the same reflection and transmission. $m_{2r} = m_{2t}$. We know $m_{2a} + m_{2r} + m_{2t} = 1$. Therefore, $m_{2r} = m_{2t} = .45$.

Lets calculate the amount of light falling per unit area in the green and blue CCDs. Light intensity is a function of inverse square distance from source. The effective distance between the pinhole and the CCDs is obtained by tracing the path through the mirrors. Thus, the red CCD is distance 2 and the other CCDs are at a distance 4 from the pinhole.

Assuming there are no mirrors in between the pinholes and the CCDs, incident light per unit area on the green CCD ,l_G, is given by

$$l_G = \frac{K}{4^2} = \frac{K}{16} \tag{1}$$

where K is some constant (We dont really need to calculate the exact value for this question). Since the blue CCD is at the same distance from the pinhole as the red CCD, light per unit area on the blue CCD, l_B, is also given by the same value:

$$l_B = \frac{K}{4^2} = \frac{K}{16} \tag{2}$$

We can now consider the effect of the mirrors. Since light falling on the blue CCD is reflected from m_1 and transmitted from m_2. Therefore, the actual amount of light per unit area of the blue CCD, L_B is given by

$$L_B = l_B * m_{1r} * m_{2t} = \frac{K}{16} * m_{1r} * m_{2t} \tag{3}$$

Similarly, light falling on the red CCD, L_R is:

$$L_R = l_R * m_{1t}\frac{K}{2^2} * m_{1t} = \frac{K}{4} * m_{1t} \tag{4}$$

(B) Note that since the blue CCD is twice as far from the pinhole as the red CCD, the central part of the red CCD which corresponds to the full blue CCD will be half the blue CCD's size in both row and column dimensions. Here size refers to the number of pixels. The pinhole camera construction can be visualized as in figure 1. Notice that the by observing the similar triangles in the figure, that $\frac{a}{b} = \frac{c}{d}$. This imples that $a = b * \frac{c}{d}$. In our example, $c = 2$ and $d = 4$. Therefore, $a = \frac{b}{2}$.

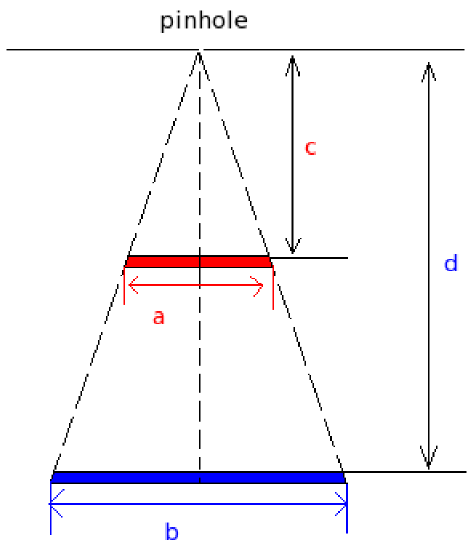

Figure 1: CCD size illustration

In order to make the relevant red CCD sub-image and blue CCD image the same size, each pixel in the red CCD will have to be copied to 4 pixels in the resized red image. If we use the resize procedure discussed in class, we copy each pixel intensity in the small image into many locations in the larger image. By making the light per unit area in the red CCD (L_R) and the blue CCD (L_B) equal, the above resize procedure will result in the required red CCD image (see figure 2).

Making light per unit area in the red and blue CCDs equal:

$$L_R = L_B \quad (5)$$

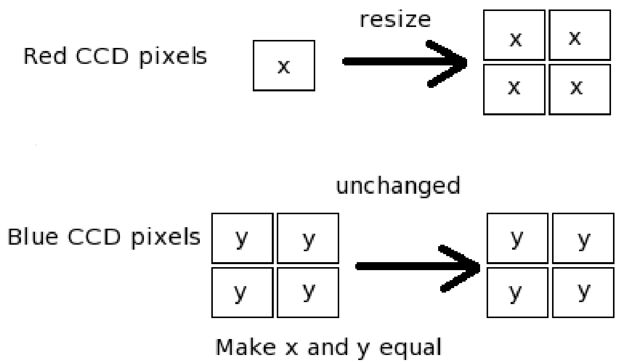

Figure 2: Red CCD resize illustration

$$\begin{aligned} &L_R = L_B \\ &\Rightarrow \frac{K}{4} * m_{1t} = \frac{K}{16} * m_{1r} * m_{2t} \\ &\Rightarrow m_{1t} = m_{1r} * m_{2t} * \frac{1}{4} \\ &\Rightarrow m_{1t} = m_{1r} * \frac{.45}{4} \end{aligned} \tag{6}$$

We know $m_{1a} + m_{1r} + m_{1t} = 1$

$$\begin{aligned} &.1 + m_{1r} + \frac{.45}{4} * m_{1r} = 1 \\ &\Rightarrow \frac{4.45}{4} * m_{1r} = .9 \\ &\Rightarrow m_{1r} = .9 * \frac{4}{4.45} = .809 \end{aligned} \tag{7}$$

Therefore, SOLUTION:
$m_{1a} = .1, m_{1r} = .809, m_{1t} = .091$
$m_{2a} = .1, m_{2r} = .45, m_{2t} = .45$