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Towards Stability Analysis of Data Transport Mechanisms: a Fluid Model and an Application

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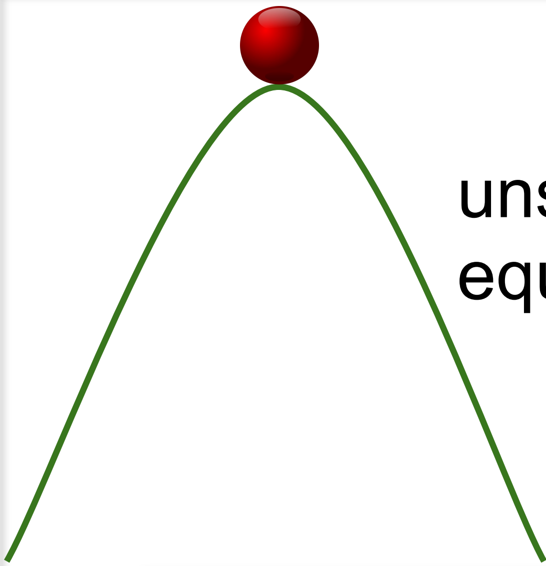
Background

- Stability well-understood: TCP Reno, STCP.
- Less so: CUBIC, H-TCP.
  - Reason: **lack of a suitable modeling framework**.

Contributions of this Work

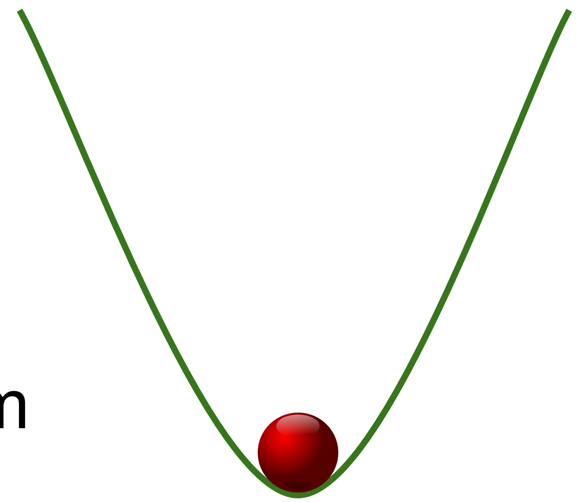
- New modeling framework applicable to wide variety of loss-based protocols.
- Application to TCP CUBIC.
  - Result: CUBIC is **locally asymptotically stable**.
- Simulation framework to test and validate the models.

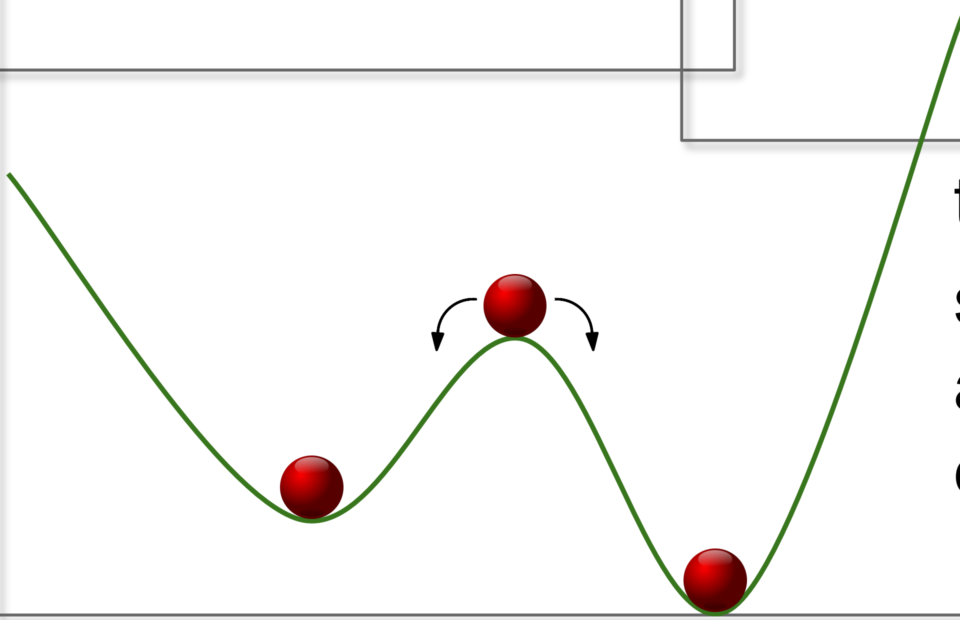
Brief Introduction to Stability



unstable
equilibrium

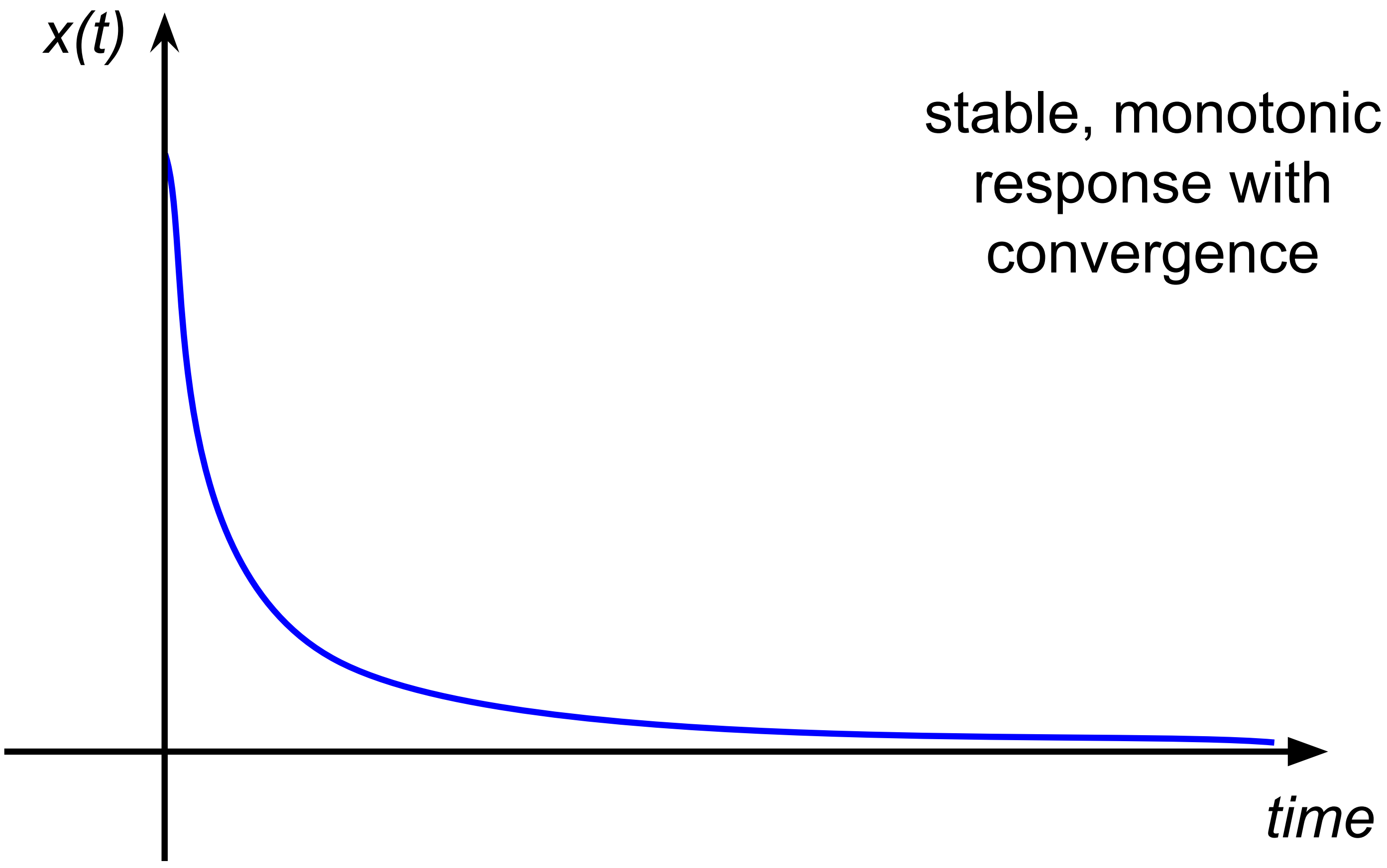
global
stable
equilibrium



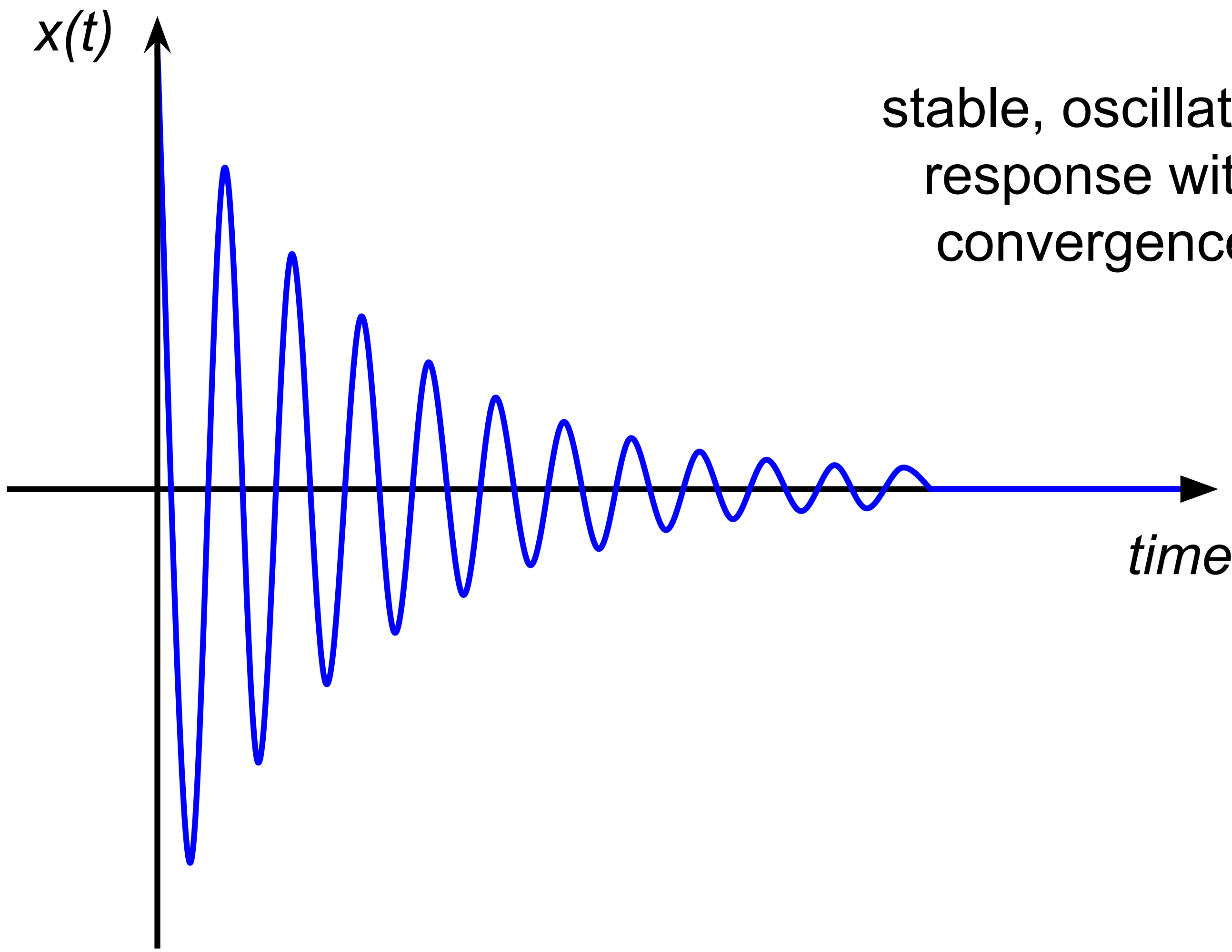


two local
stable equilibria
and one unstable
equilibrium

Stability: Difference in Responses

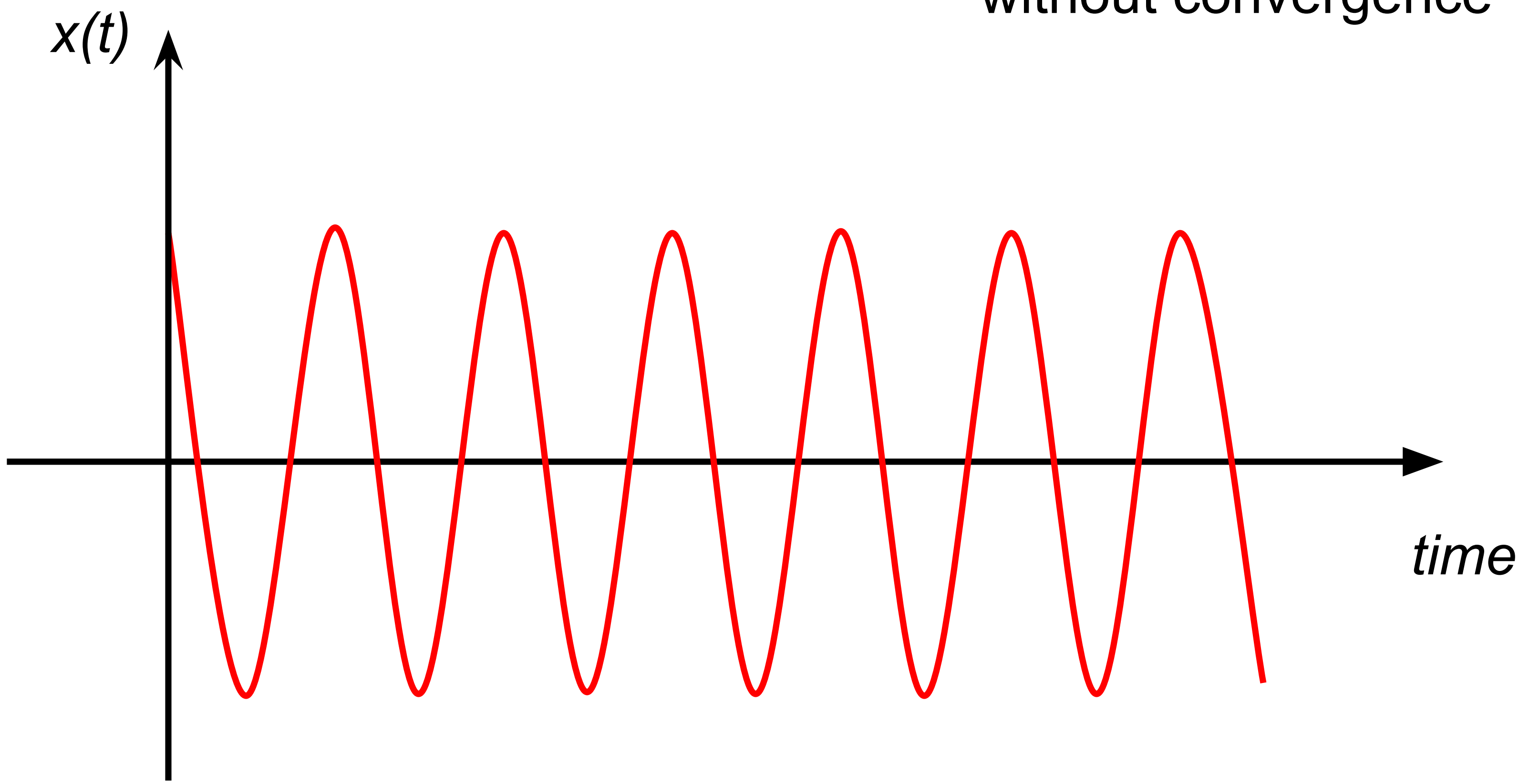


Stability: Difference in Responses

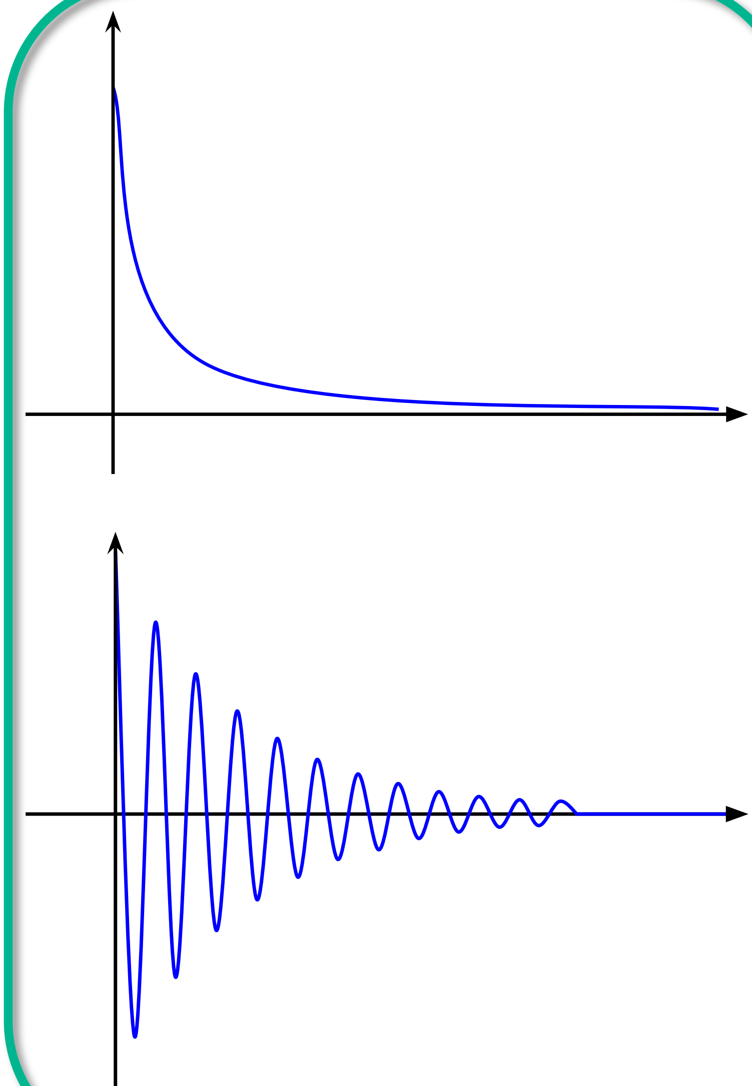


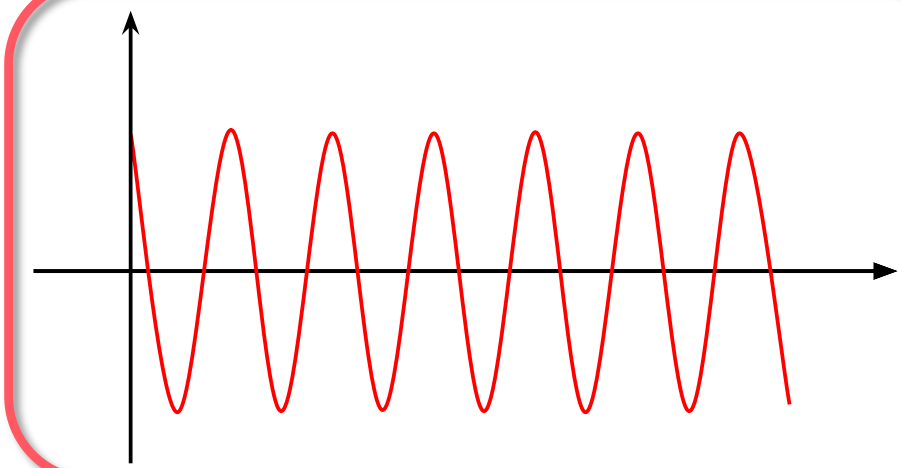
Stability: Difference in Responses

oscillatory response
without convergence



Stability: Difference in Responses

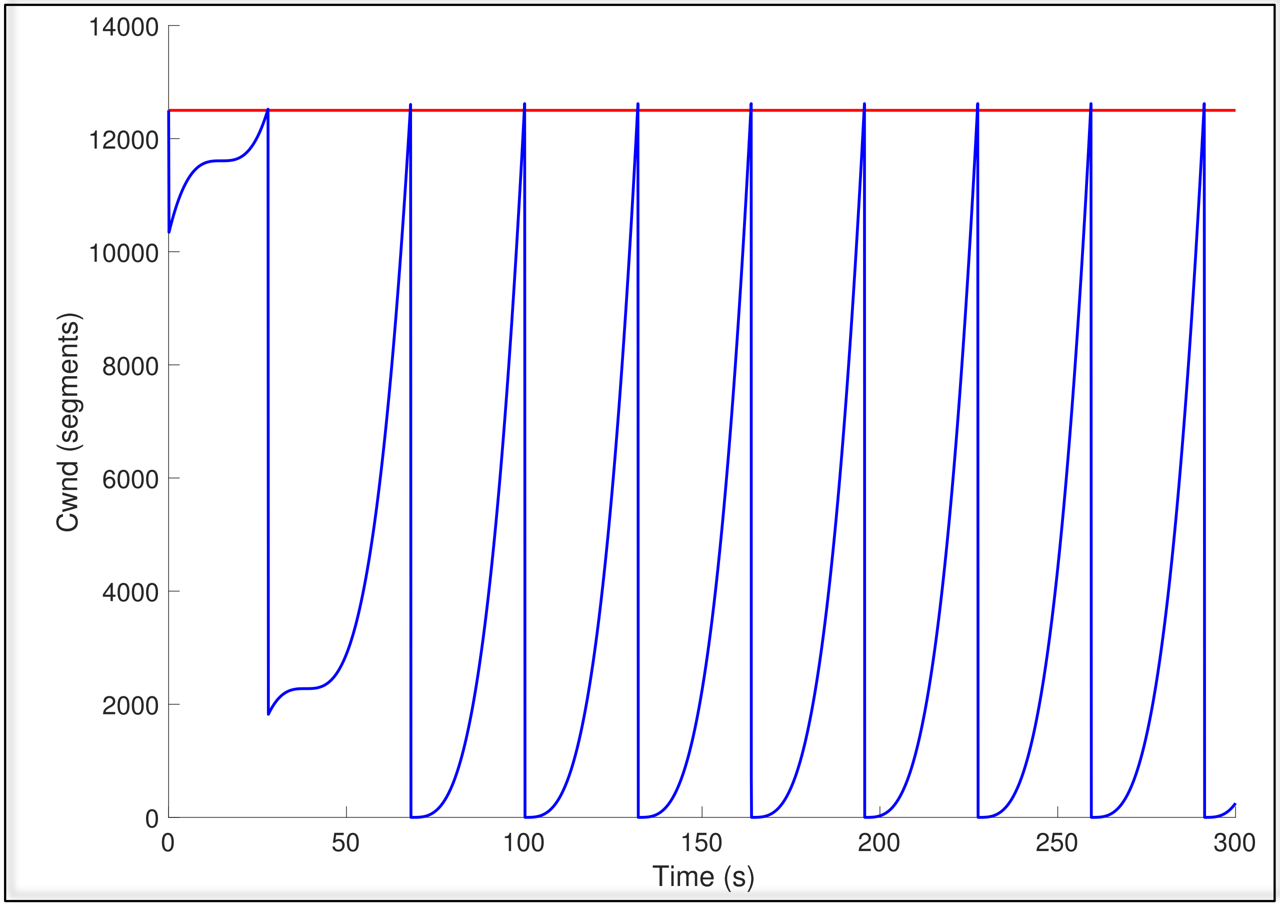




Undamped oscillation is **undesirable** behavior.

Convergence to a fixed point is **desirable** behavior.

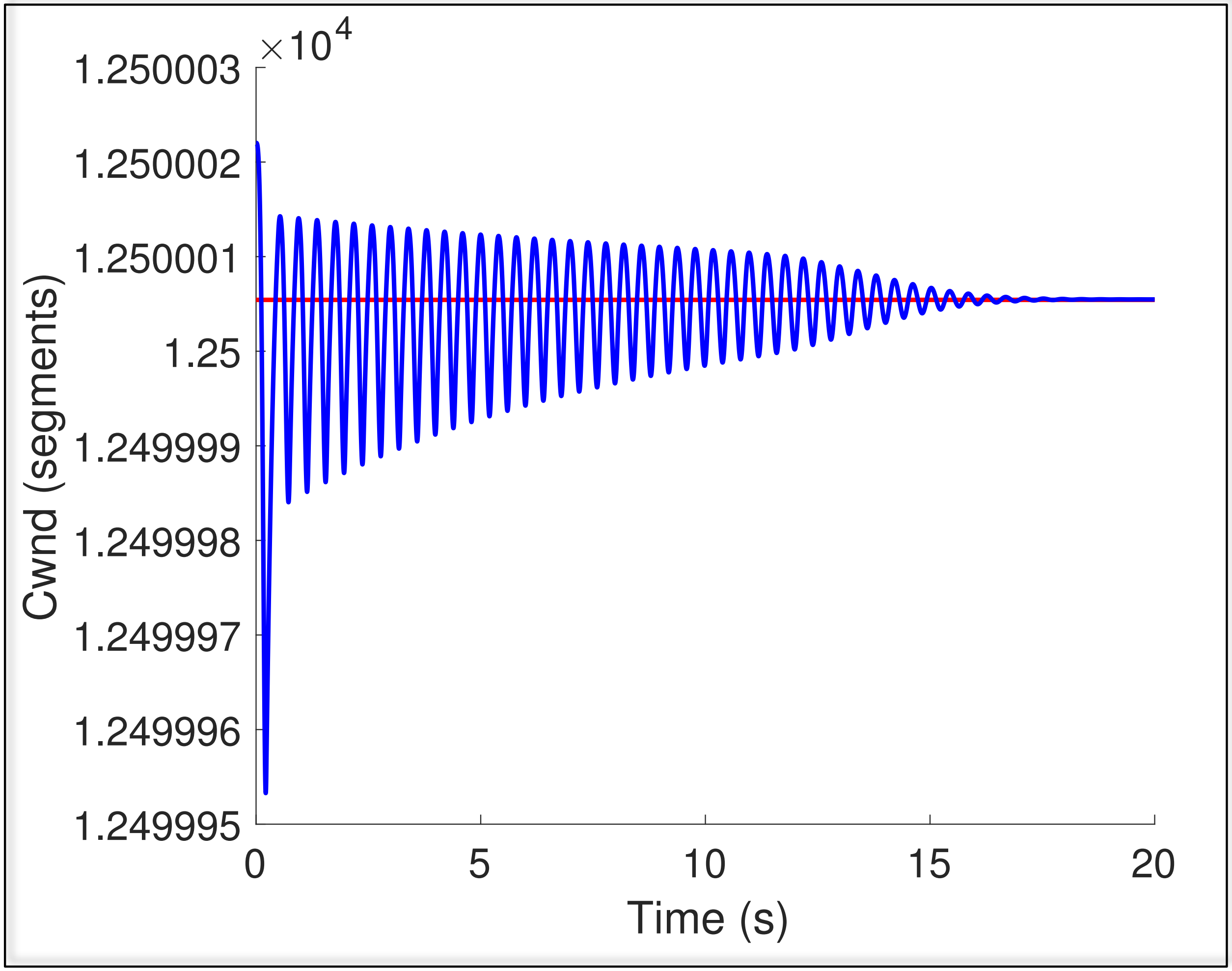
Instability with TCP CUBIC



Fluid model simulation.
Link capacity of 1 Gbps,
delay of 100 ms.

This choice of initial conditions leads to unstable behavior.

Stability with TCP CUBIC



Fluid model simulation.
Link capacity of 1 Gbps,
delay of 100 ms.

This choice of initial conditions leads to stable behavior.

The Lesson

TCP CUBIC can be ill-behaved.

In general, more deviation from fixed-point → more instability.

Impact on performance metrics like bandwidth utilization.

Effective modeling → conditions for stability → ensure efficient operation of the protocol.

Previous Work

- Misra *et al*. **"Fluid-based Analysis of a Network of AQM Routers Supporting TCP Flows with an Application to RED."** *ACM SIGCOMM 2000.*

$$\frac{dW(t)}{dt} = \frac{1}{\tau} - \frac{W(t)}{2}\lambda(t-\tau) \longleftarrow (*)$$

- Hollot *et al*. **"A Control Theoretic Analysis of RED."** INFOCOM 2001.
  - Analyze system above, present design guidelines for stable AQM operation.

- Liu *et al*. **"Fluid Models and Solutions for Large-Scale IP Networks."** ACM/SigMetrics 2003.
  - Uses (*) as a starting point. Model a network of AQM routers. Obtain transient behavior of average queue lengths, packet loss probabilities, latencies.

Definitions

Term	Definition
C	per-flow capacity
τ	link delay
$W_{\max}(t)$	the size of the *cwnd* immediately before loss
$s(t)$	the time elapsed since last loss
$W(t)$	the *cwnd as a function of time*
$p(t)$	a loss probability function
$\hat{x}$	fixed-point value of x

TCP CUBIC: Definition

Congestion window function is given by

$$W(t) = c\left(s(t) - \sqrt[3]{\frac{W_{\max}(t)b}{c}}\right)^3 + W_{\max}(t)$$

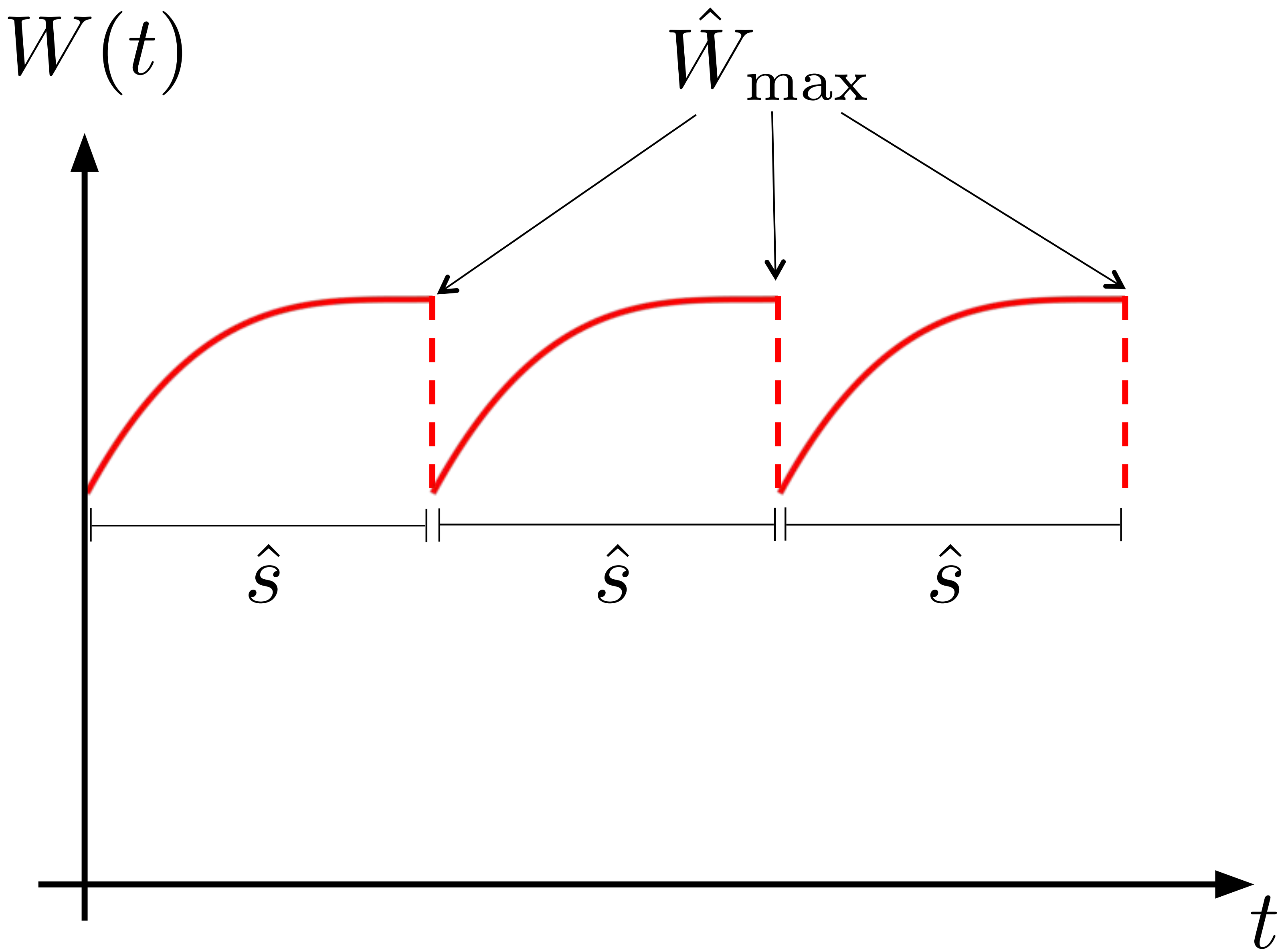
c - scaling factor,

b - multiplicative decrease constant,

$s(t)$ - elapsed time since last loss,

$W_{\max}(t)$ - size of *cwnd* immediately before last loss.

TCP CUBIC in Steady State



Modeling CUBIC

- Reno: AIMD

$$\frac{dW(t)}{dt} = \frac{1}{\tau} - \frac{W(t)}{2}\lambda(t-\tau)$$

- Scalable TCP: MIMD
- CUBIC:

$$W(t) = c\left(\boldsymbol{s(t)} - \sqrt[3]{\frac{\boldsymbol{W_{\max}(t)}b}{c}}\right)^3 + \boldsymbol{W_{\max}(t)}$$

New Approach

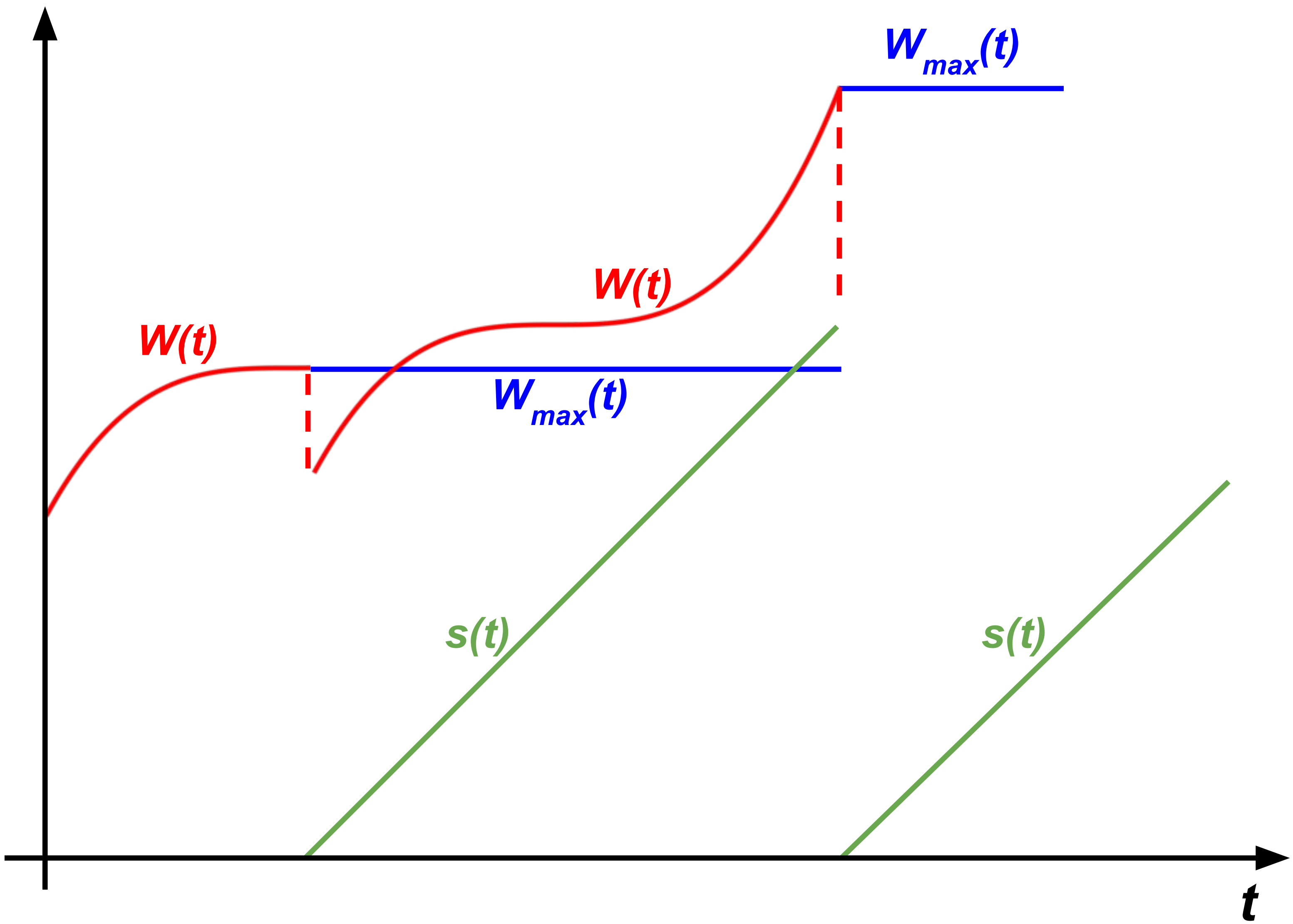
Observation: loss-based protocols have in common:

- max *cwnd* before loss, $W_{max}(t)$
- time since last loss, $s(t)$

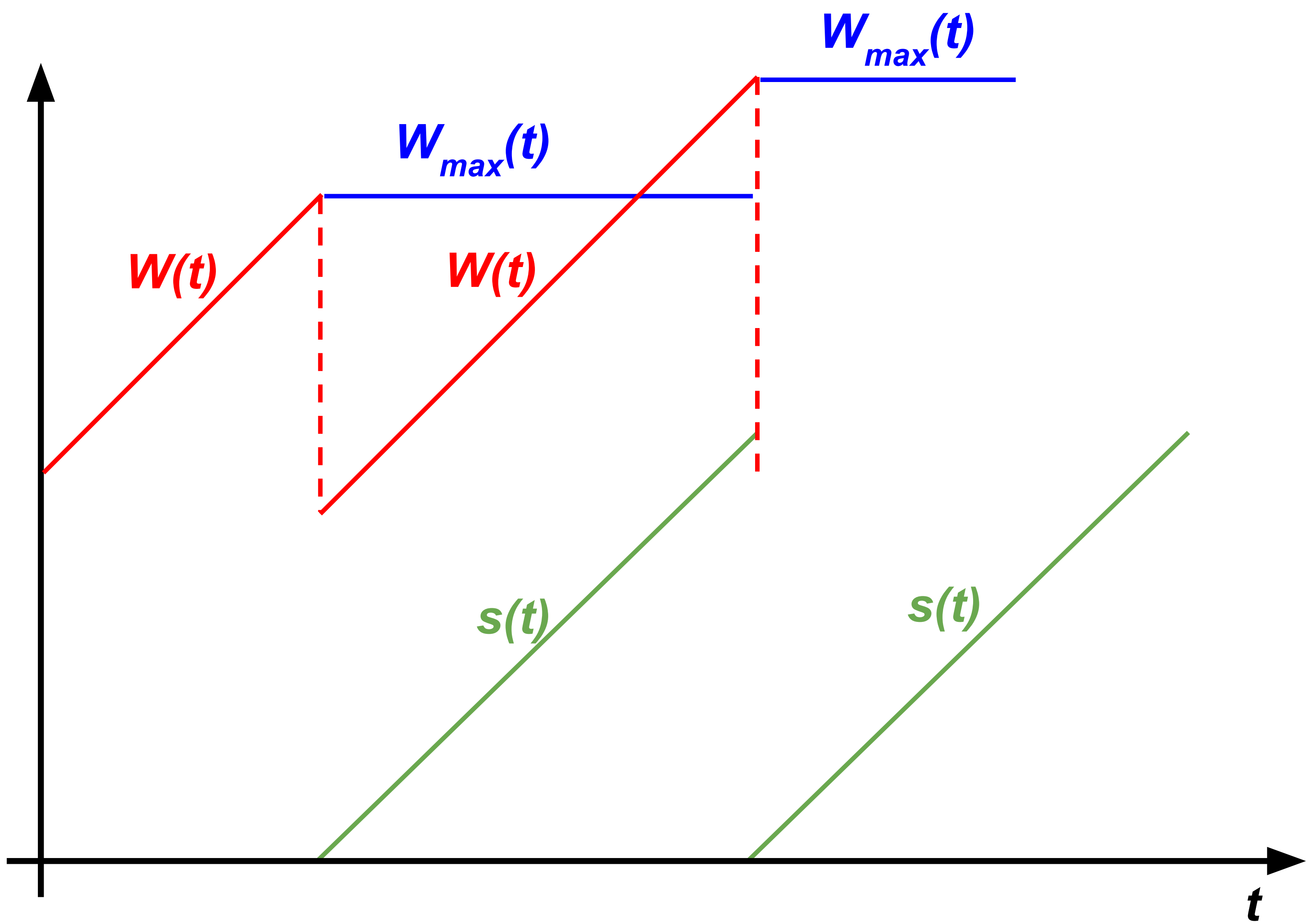
Derive DEs for

instead of for W(t)!

Example: TCP CUBIC



Example: TCP Reno



New Model

System of differential equations

$$(1) \quad \frac{dW_{\max}(t)}{dt} = -(W_{\max}(t) - W(t))\frac{W(t-\tau)}{\tau}p(t-\tau)$$

$$(2) \quad \frac{ds(t)}{dt} = 1 - s(t)\frac{W(t-\tau)}{\tau}p(t-\tau)$$

Loss probability function

$$(3) \quad p(t) = \max\left(1 - \frac{C\tau}{W(t)}, 0\right)$$

New Model: First DE

change in maximum congestion window size

$$\frac{dW_{\max}(t)}{dt} = -(W_{\max}(t) - W(t))\frac{W(t-\tau)}{\tau}p(t-\tau)$$

delayed by one round trip time, since loss occurs at a congestion point, not at the source

packet loss rate

New Model: Second DE

time since last loss grows by one time unit, and is reset to zero upon new loss

$$\frac{ds(t)}{dt} = 1 - s(t)\frac{W(t-\tau)}{\tau}p(t-\tau)$$

delayed by one round trip time, since loss occurs at a congestion point, not at the source

packet loss rate

Application to CUBIC

$$(1)\quad \frac{dW_{\max}(t)}{dt} = -(W_{\max}(t) - W(t))\frac{W(t-\tau)}{\tau}p(t-\tau)$$

$$(2)\quad \frac{ds(t)}{dt} = 1 - s(t)\frac{W(t-\tau)}{\tau}p(t-\tau)$$

System of differential equations

+

$$(3)\quad p(t) = \max\left(1 - \frac{C\tau}{W(t)}, 0\right)$$

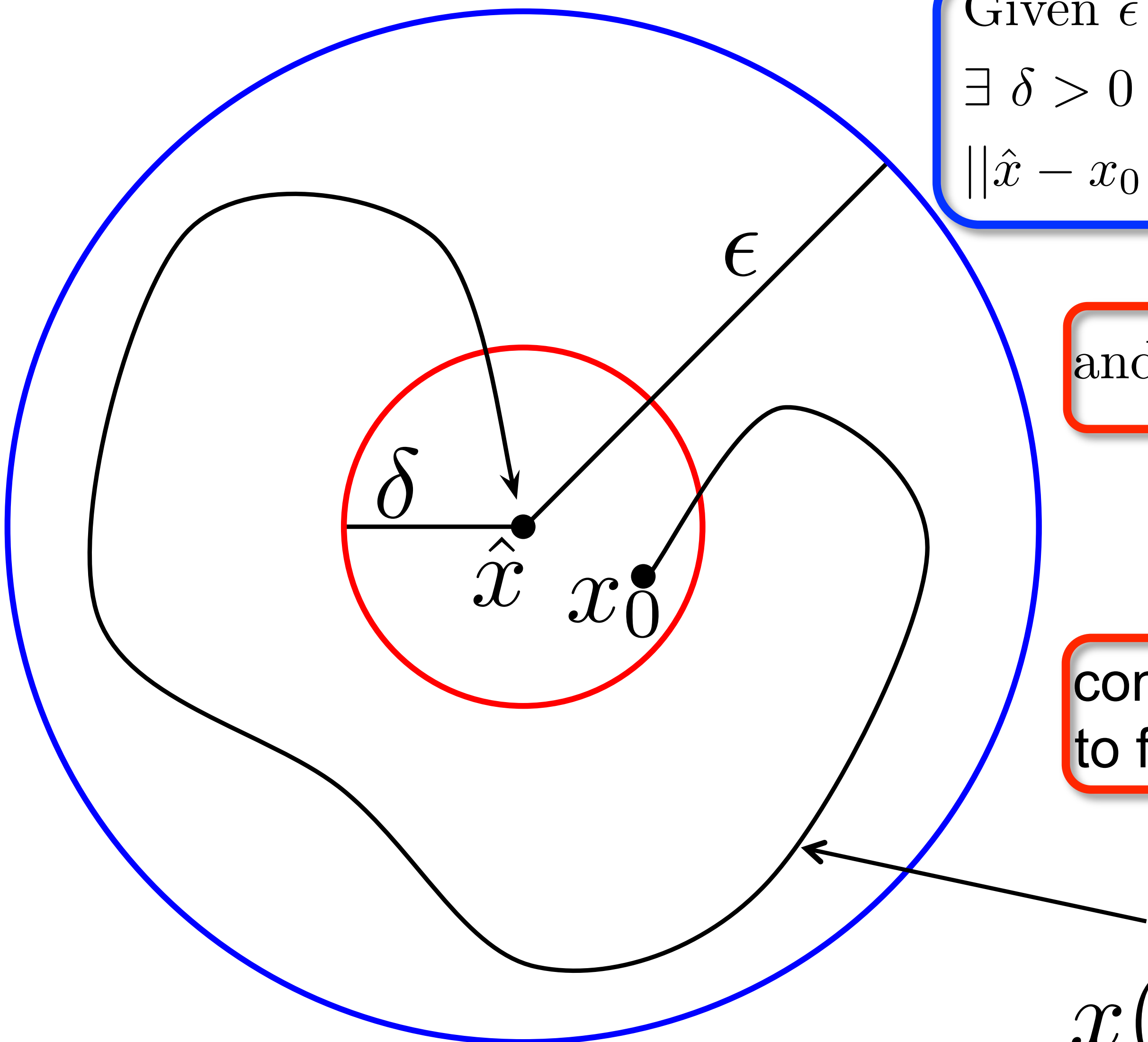
Loss probability function

+

$$W(t) = c\left(s(t) - \sqrt[3]{\frac{W_{\max}(t)b}{c}}\right)^3 + W_{\max}(t)$$

Congestion window function

Asymptotic Stability



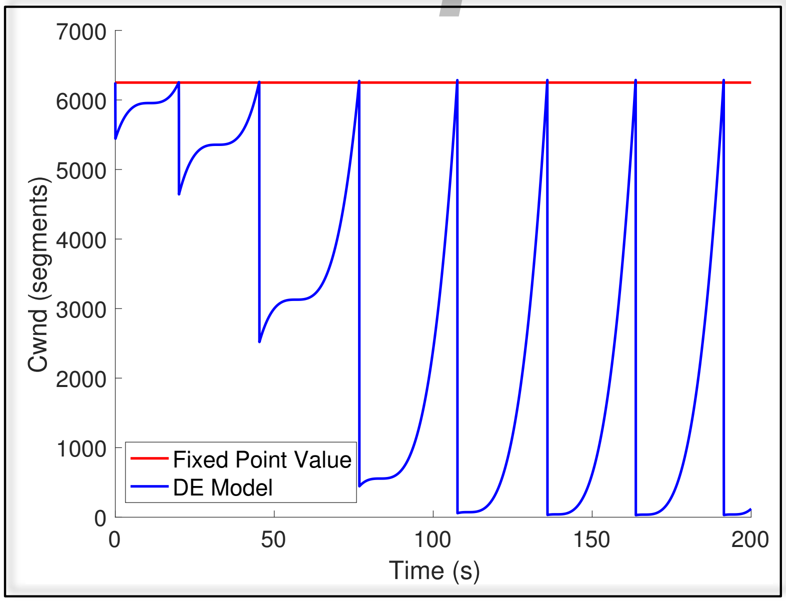
Given $\epsilon > 0$, for any $t > 0$,
$\exists\ \delta > 0$ s.t.
$||\hat{x} - x_0|| < \delta \implies ||\hat{x} - x(t)|| < \epsilon$

and $\lim_{t \to \infty} ||\hat{x} - x(t)|| = 0.$

convergence to fixed point + Lyapunov stability

$$x(t) = W(t)$$

Examples: Fluid Model



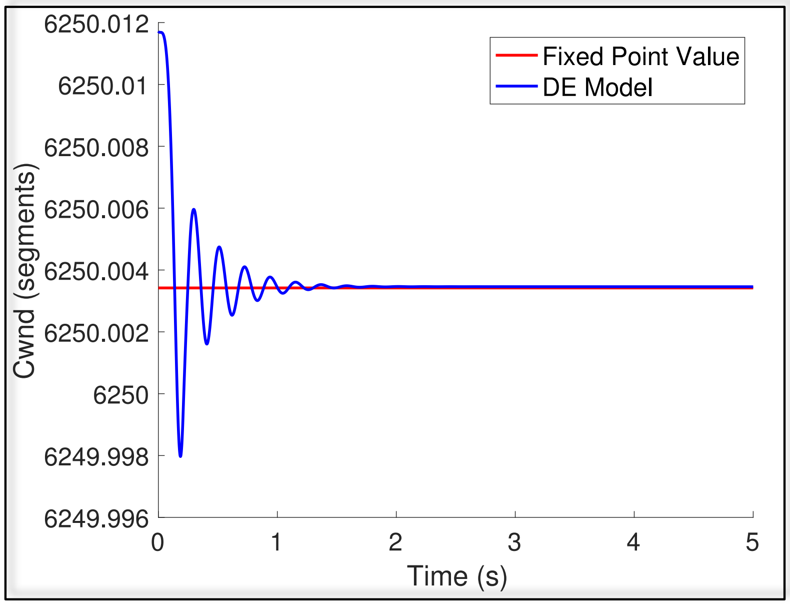
$C = 1$ Gbps

$\tau = 50$ ms

$|\hat{W} - W_0| = 1.66$ segments

$|\hat{s} - s_0| = 4.4$ seconds

initial conditions deviate too far from fixed point



$C = 1$ Gbps

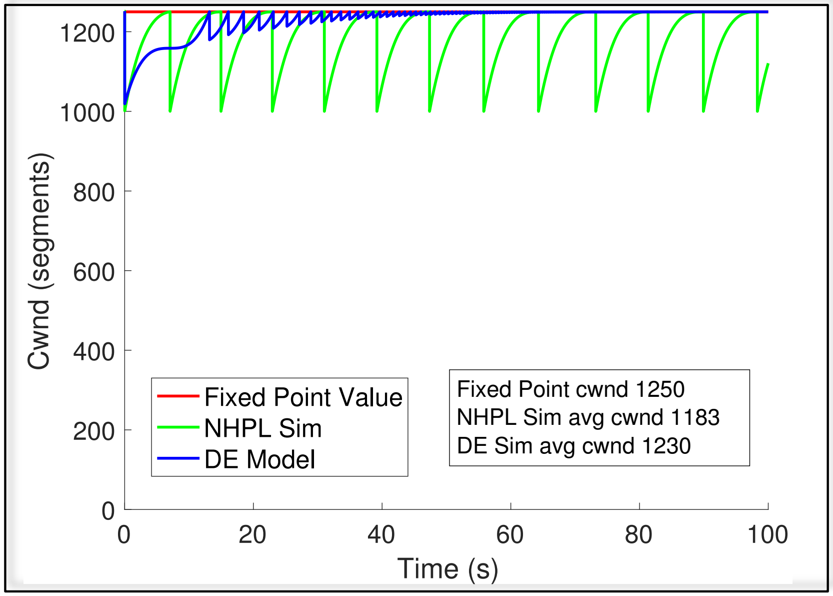
$\tau = 50$ ms

$|\hat{W} - W_0| = 0.008$ segments

$|\hat{s} - s_0| = 0.044$ seconds

initial conditions yield asymptotically stable response

Model Validation



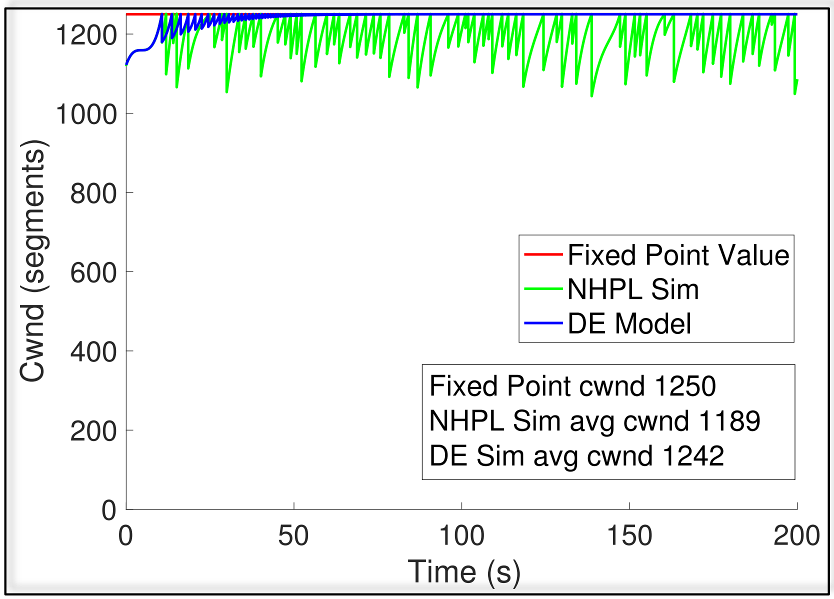
$C = 1$ Gbps

$\tau = 10$ ms

one flow

NHPL – **N**on-**H**omogeneous **P**oisson **L**oss Simulation

Possible application of what we learned: fixed point value can be used to choose initial *ssthresh*.



$C = 1$ Gbps

$\tau = 10$ ms

20 flows

Summary

- New modeling framework consisting of a set of differential equations, loss probability function, and congestion window or sending rate function.
- Model used to analyze TCP CUBIC and establish that it is locally asymptotically stable.
- New lightweight simulation framework generalizable to a variety of protocols.
  - Used this to validate the fluid model.

See Vardoyan *et al. arXiv:1801.02741, Jan 2018* for full proofs, convergence result, detailed description of simulation framework, and more...

Thank you!

Questions?

Backup Slides

Model Equivalence

$$W(t) = \frac{W_{\max}(t)}{2} + \frac{s(t)}{\tau}$$

$$\dot{W} = \frac{\dot{W}_{\max}}{2} + \frac{\dot{s}}{\tau}$$

$$\dot{W} = \frac{1}{\tau}\left(1 - s\frac{W_\tau}{\tau}p_\tau\right) + \frac{1}{2}\left((W - W_{\max})\frac{W_\tau}{\tau}p_\tau\right)$$

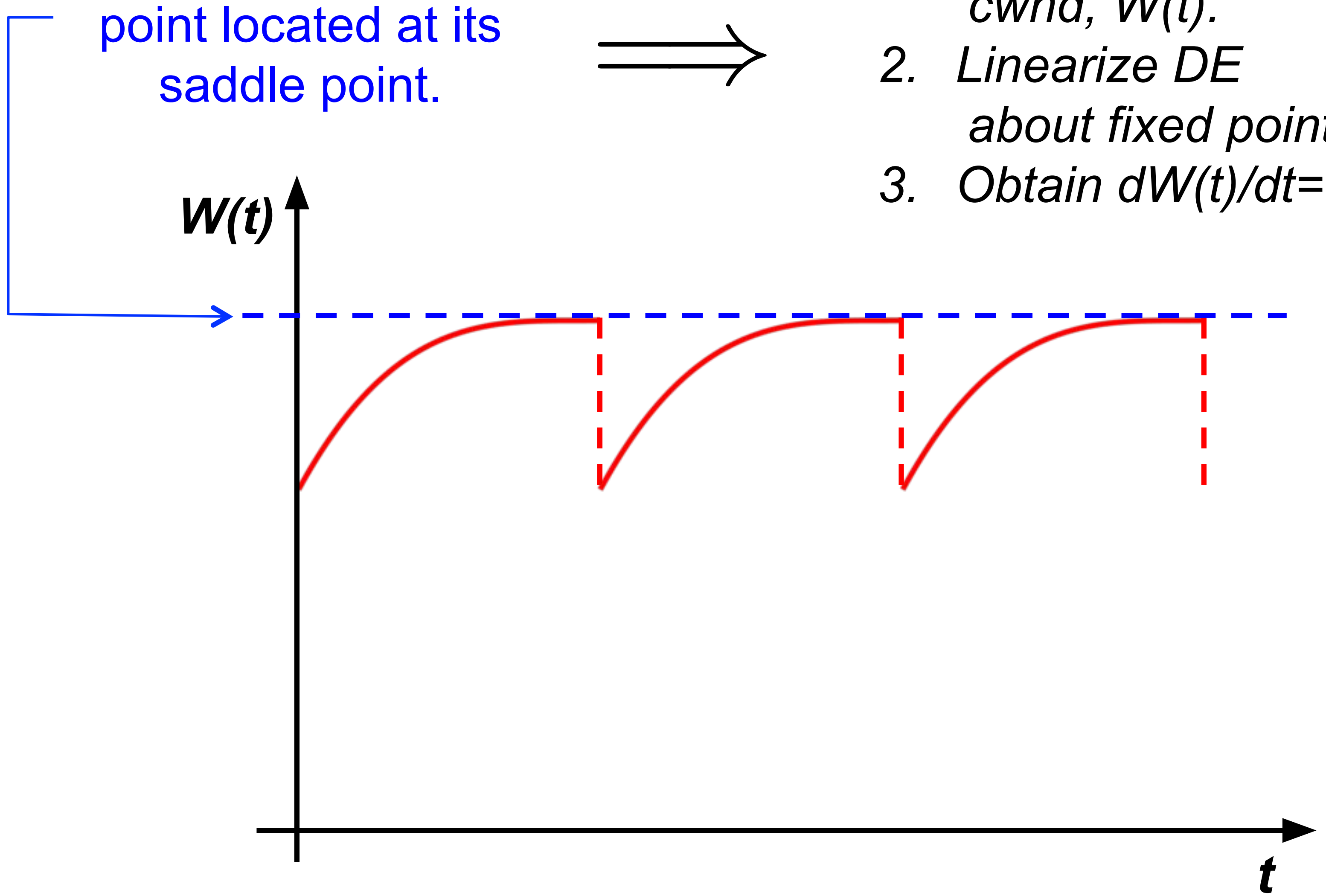
$$\begin{aligned}\dot{W} &= \frac{1}{\tau}\left(1 - s\frac{W_\tau}{\tau}p_\tau\right) + \frac{1}{2}\left(\left(W - 2\left(W - \frac{s}{\tau}\right)\right)\frac{W_\tau}{\tau}p_\tau\right)\\ &= \frac{1}{\tau} - s\frac{W_\tau}{\tau^2}p_\tau - \frac{W}{2}\frac{W_\tau}{\tau}p_\tau + s\frac{W_\tau}{\tau^2}p_\tau\\ &= \frac{1}{\tau} - \frac{W}{2}\frac{W_\tau}{\tau}p_\tau.\end{aligned}$$

Initial Attempt to Model CUBIC

CUBIC has a fixed point located at its saddle point.

$\Longrightarrow$

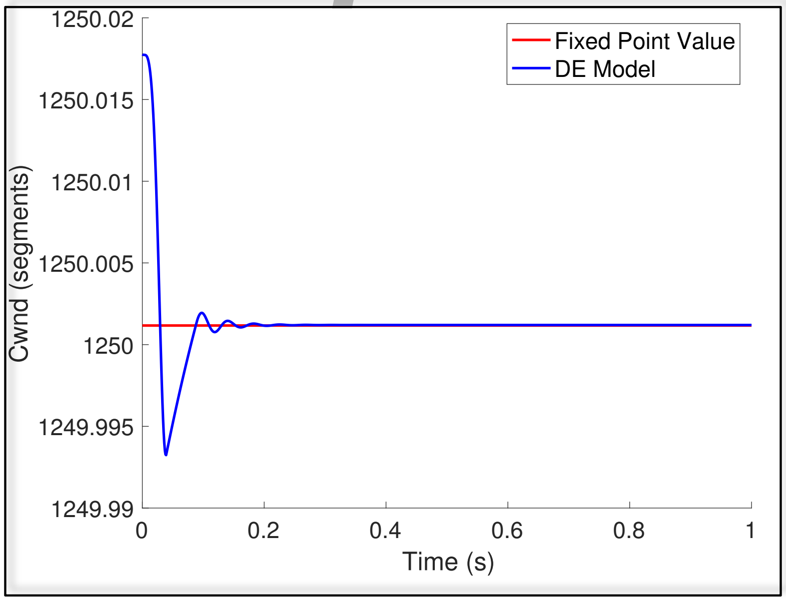
1. Derive DE for *cwnd, W(t).*
2. *Linearize DE about fixed point.*
3. *Obtain dW(t)/dt=0.*

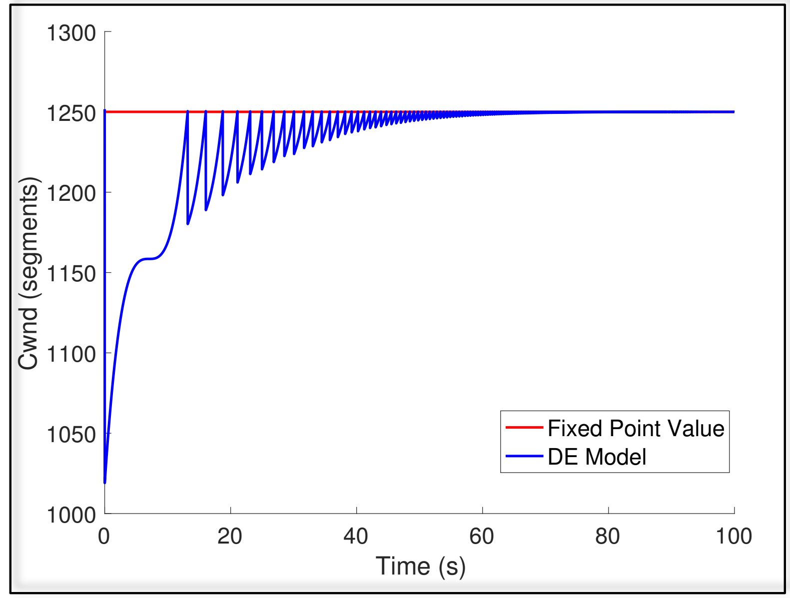


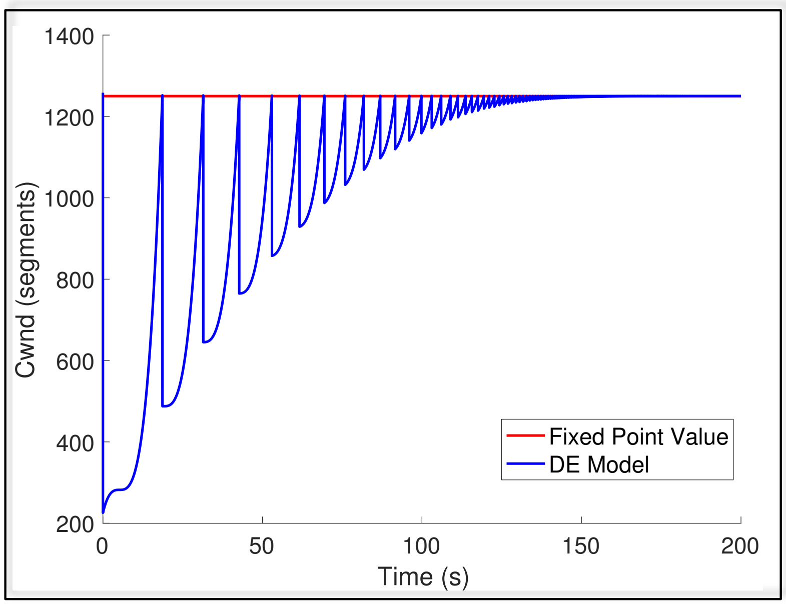
Current and Future Work

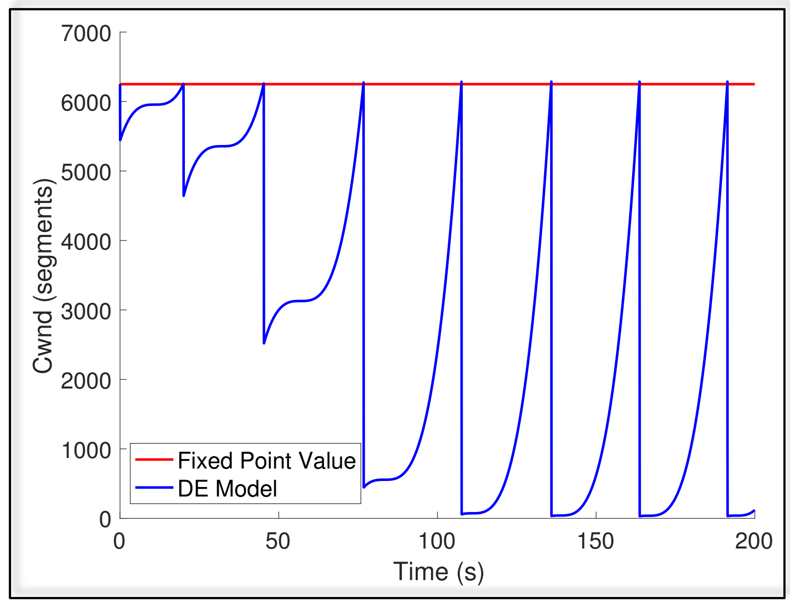
- Developed a linearizable version of CUBIC.
  - Simulations show that while this controller is more responsive, it is also less stable.
- Application of fluid model to H-TCP.
  - Conditions derived for stability.
  - Simulations show that H-TCP in general less stable than CUBIC.

Examples


Fixed Point Value
DE Model
Cwnd (segments)
1250.02
1250.015
1250.01
1250.005
1250
1249.995
1249.99
0
0.2
0.4
0.6
0.8
1
Time (s)


Fixed Point Value
DE Model
Cwnd (segments)
1300
1250
1200
1150
1100
1050
1000
0
20
40
60
80
100
Time (s)


Fixed Point Value
DE Model
Cwnd (segments)
1400
1200
1000
800
600
400
200
0
50
100
150
200
Time (s)


Fixed Point Value
DE Model
Cwnd (segments)
7000
6000
5000
4000
3000
2000
1000
0
0
50
100
150
200
Time (s)

Model Validation