CS311:	**Homework 10**	Due in class, 11:15 am, Tuesday, Dec. 2, 2008

1. [25 pts.] Do problem 7.29, page 230: a linear program for Hollywood. Part (a) asks you to write the given Hollywood optimization problem as a linear program in which all the variables are constrained to be either 0 or 1. Part (b) asks you to rewrite this as an ordinary linear program and to prove that there is an optimal solution to your linear program such that every variable has value 0 or 1. A hint on how to do this is to consider an optimal solution, s, with the fewest number of non-integer values amongst all optimal solutions. Now consider the minimum nonintegral value, v, taken on by any of the variables. Show that all the variables of value v can be increased to 1 without decreasing the objective function nor violating any of the constraints. Look at a few small examples until you get the idea why this will hold.

2. [25 pts.] Do problem 7.30, page 230: prove Hall's Theorem: Let $H = (V, E)$ be an undirected bipartite graph with n boys and n girls, $V = B \cup G$. Then H has a perfect matching iff H has the property that any subset $S \subseteq B$ has at least $|S|$ girls as neighbors.

   Hint: first prove that if there is a perfect matching, then H has Hall's property.

   Next suppose that there is no perfect matching. Consider the network flow problem corresponding to H's matching problem. Since there is no perfect matching, there must be a cut with capacity at most k for some $k < n$. Use this to show that Hall's property does not hold.

3. [25 pts.] Do problem 8.1, page 264: Show that if TSP can be solved in polynomial time, then so can TSP-OPT. Hint: use binary search.

4. [25 pts.] Do problem 8.5, page 265: give reductions from 3D matching to SAT and from Hamilton (Rudrata) Cycle to SAT.