

Note: I will hand out model solutions to this last homework set at the discussion section Thursday, Dec. 11, so I cannot accept late homeworks after that.

1. [25 pts.] Do problem 8.9, page 266: prove that the Hitting-Set problem is NP complete. [Hint: To do this you must argue that Hitting-Set is in NP, and give a polynomial-time reduction from some NP complete problem to Hitting-Set. I suggest that you reduce 3-SAT to Hitting-Set. Suppose that your input is a 3-CNF formula, φ , with m boolean variables, x_1, \dots, x_m , and r clauses, C_1, \dots, C_r . You want to compute an instance of the Hitting-Set problem, $f(\varphi)$ such that,

$$\varphi \in 3\text{-SAT} \quad \Leftrightarrow \quad f(\varphi) \in \text{Hitting-Set} .$$

I suggest that you make the problem $f(\varphi)$ have $n = m + r$ sets to be hit.]

2. [25 pts.] Do problem 8.10, page 266: a, c, and f: show that these three problems are in NP and show that they are NP complete by explaining how they are generalizations of known NP complete problems.
3. [25 pts.] Do problem 5.33, page 154: show how to implement the given greedy algorithm (called “Stingy” in the text, §5.3) in linear time.
4. [25 pts.] Do problem 9.1, page 293: show that the backtracking algorithm for 2-SAT, with the following heuristic runs in polynomial time. The heuristic is: always choose a subproblem (CNF formula) that has a clause that is as small as possible and expand it along a variable that appears in this small clause.