CS311:	**Homework 6**	Due in class, 11:15 am, Tuesday, Oct. 21, 2008

Note: In order to get us back to homeworks due on Tuesdays, this homework will be graded, thought of as maximum grade 75 + 25 points extra credit. Think of 10 points of the second question as extra credit and 15 points of the third question as extra credit.

1. [25 pts.] Question 3.18, page 98: linear-time preprocessing of a tree so that the ancestor query can be executed in constant time.

2. [35 pts.] Question 4.13, p. 121: looking for paths minimizing the maximum edge length along the path.

3. [40 pts.] An undirected graph, $G = (V, E)$, is called **bi-partite** if V can be partitioned into two disjoint sets, $V = V_1 \cup V_2$, $V_1 \cap V_2 = \emptyset$, such that every edge goes between the two parts, i.e., $E \subseteq V_1 \times V_2$.

   An undirected graph, $G = (V, E)$ is k-colorable, iff there exists a coloring map, $c : V \to \{1, \ldots, k\}$ that colors each vertex one of k colors in such a way that no two adjacent vertices are the same color, i.e., $E(i, j) \to (c(i) \neq c(j))$.

   (a) Prove that a graph is bipartite iff it is two colorable

   (b) Give a linear-time algorithm to test if a graph is 2-colorable. If the answer is "yes", the algorithm should produce the 2-coloring. If the answer is "no", the algorithm should produce an odd length cycle.

   (c) Prove that a graph is 2-colorable iff it has no odd length cycle.