Some of you asked for some notes about induction. This relates to Problem 1 on HW 2, where I suggest that you prove the correctness of the text's division algorithm by induction on $n = \lfloor \log(x+1) \rfloor$.

**Principle of Mathematical Induction** Let $P(n)$ be a proposition that is true or false for each given natural number, $n \in \mathbf{N}$. Suppose that the following two facts hold:

**base case:** $P(0)$

**inductive case:** $\forall r \in \mathbf{N}(P(r) \rightarrow P(r+1))$

Then, we can conclude that for all $n \in \mathbf{N}$, $P(n)$ holds.

**Example 1** A simple use of Mathematical Induction is to prove that for all postive reals, $a \neq 1$, and for all $n \in \mathbf{N}$, the formula $\displaystyle\sum_{i=0}^{n} a^i = \frac{(a^{n+1}-1)}{(a-1)}$ always holds.

**Proof:** Let $G(n)$ be the statement $\displaystyle\sum_{i=0}^{n} a^i = \frac{(a^{n+1}-1)}{(a-1)}$. We prove by induction that $G(n)$ holds for all $n \in \mathbf{N}$.

**base case:** $G(0)$ says that $\displaystyle\sum_{i=0}^{0} a^i = \frac{(a^{0+1}-1)}{(a-1)}$. The summation on the left has one term namely $a^0 = 1$. The term on the right is $\frac{(a-1)}{(a-1)} = 1$, so $G(0)$ is true.

**inductive case:** We may assume inductively that $G(r)$ holds, where $r$ is a fixed, arbitrary natural number, i.e, $\displaystyle\sum_{i=0}^{r} a^i = \frac{(a^{r+1}-1)}{(a-1)}$.

We want to prove that $G(r+1)$ holds.

Note that $\displaystyle\sum_{i=0}^{r+1} a^i = \sum_{i=0}^{r} a^i + a^{r+1}$.

By our inductive hypothesis, we thus have that $\displaystyle\sum_{i=0}^{r+1} a^i = \frac{(a^{r+1}-1)}{(a-1)} + a^{r+1}$.

But note that,

$$\begin{aligned} \frac{(a^{r+1}-1)}{(a-1)} + a^{r+1} &= \frac{(a^{r+1}-1+(a-1)a^{r+1})}{(a-1)} \\ &= \frac{(a^{(r+1)+1}-1)}{(a-1)} \end{aligned}$$

Thus, as desired, $G(r+1)$ holds.

Since we have proved the base case and the inductive case, it follows by the principle of mathematical induction that $G(n)$ holds for all $n \in \mathbf{N}$ □

Hope this helps. I'm off to a wedding in Seattle, back Sunday night. Please feel free to use Creidieki's office hours on Friday and mine on Monday.