CS513:	**Homework 1**	Due in class, Sept. 24, 2009

Cooperation: Students should talk to each other about the subject matter of this class and help each other. It is fine to discuss the problems and ask questions about them. I encourage such questions in class and office hours as well as elsewhere. However, there is a line past which you must not go, e.g., sharing or copying a solution is not okay and could result in failure. If a significant part of one of your solutions is due to someone else, or something you've read then **you must acknowledge your source!** Failure to do so is a serious academic violation, likely to result in failure of the course or worse. Furthermore, **all solutions must be written by yourself, in your own words**. You may get an idea from somewhere or someone and acknowledge that, but you must still understand it and explain it yourself. A copied solution, even with the source acknowledged will be considered plagiarism. The exception is if it is in quotation marks and cited specifically. But in this case, don't bother because you won't get credit for quoting someone else's solution.

General Strategy for doing the homeworks: My desire for the homeworks is that they help you **understand** the concepts from lecture and readings, i.e., that these concepts not only seem believable, but you can employ them. Please read the homework early and make sure that you understand all the questions.

For almost all the problems I give, you should be able to look at a few tiny examples, and try to solve the problem on those examples. If you can do it for the tiny examples, then you are part way to giving a rule that will solve the problem in general. If you cannot solve or are confused by what the problem means on one of your small examples, then that would be a great time to ask me a question about it.

Reading Please read the whole first chapter of Schöning by the time this assignment is due. I suggest that you only read one section at a sitting and that you think about all of the exercises – assigned or not – to help make sure that you are understanding all the concepts as you read.

Problems

1. [20 pts.] Do Exercise 10, p. 13.

2. [20 pts.] Do Exercise 11, p. 13.

3. [25 pts.] Prove the following Theorem: If $\mathcal{A}$ and $\mathcal{B}$ are suitable truth assignments for propositional formula F and $\mathcal{A}$ and $\mathcal{B}$ agree on all the atomic formulas that occur in F, then $\mathcal{A}(F) = \mathcal{B}(F)$.

   Hint: you should prove this by induction on the structure of the formula F: there are two base cases:

   0. $F \in \{\top, \bot\}$.
   1. $F = A_i$, for some atomic formula, A_i.

   and two inductive cases in terms of smaller formulas E, G:

   3. $F = \neg E$.
   4. $F \in \{(E \vee G), (E \wedge G)\}$.

4. [25 pts.] Do exercise 3, p. 10: For propositional formulas $F_1, \ldots, F_k, G$, show that the following conditions are equivalent (TFAE):

   (a) $\{F_1, \ldots, F_k\} \models G$

   (b) $\models \bigwedge_{i=1}^{k} F_i \rightarrow G$

   (c) $\left(\bigwedge_{i=1}^{k} F_i\right) \wedge \neg G \in \text{UNSAT}$.

5. [10 pts.]* We saw in class that every propositional formula is equivalent to a formula in CNF and a formula in DNF. However, this does not mean that it is computationally easy to put a formula in one of these normal forms. Show that the process is exponential in the worst case. That is, show that there is an exponent $e > 1$ such that for every n, there is a propositional formula F of length less than n such F is in CNF but any equivalent formula in DNF has length greater than e^n.

* Starred problems are meant to be challenging and it is perfectly okay not to get them. I will consider a 90 on this homework **very** good.