CS513:	**Homework 3**	Due in class, Oct. 22 2009

Reading Please finish reading Section 2.5 by the time this assignment is due.

Problems

1. [20 pts.] Let $F = \forall x((\forall y Q(y,x)) \rightarrow \forall x R(z,x))$.
   (a) Transform F into an equivalent formula, F' in prenex normal form. Please show your work.
   (b) Now Skolemize F'.
2. [20 pts.] Do Exercise 64, p. 59: dual of Skolemization.
3. [20 pts.] Do Exercise 67, p 66: prove that satisfiability and validity are still undecidable for predicate logic formulas with no occurrences of function symbols. [Hint: in the presence of an equality predicate, $=$, that is guaranteed to be interpreted as true equality, show how to transform any formula, F, into an "essentally equivalent" formula, G where each function symbol f of arity k is replaced by a predicate symbol R_f of arity $k+1$. For this question, it suffices to do this with function symbols of arity one. To complete this problem you may assume that you have the equality predicate. If you have some extra time and want some extra credit, then argue why the result – that validity for predicate logic remains undecidable without function symbols – remains true without equality.]
4. [20 pts.]* Do exercise 73, p. 77: show that the notions of "semi-decidablity" and recursive enumerability (r.e.) are equivalent. A set, M, is semi-decidable if there is an algorithm that halts and answers, "yes", on exactly all the elements of M. On non-elements of M it may halt and say "no" or never halt. A set M is r.e. if it is empty or if it can be written as $M = \{f(1), f(2), \ldots\}$ where f is a total and computable function. By total, I mean that it halts and computes an answer on all inputs $1, 2, \ldots$.
5. [20 pts.]* Do exercise 71, page 69: argue that every complete and axiomatizable theory is decidable.