

Reading Please read Sections 2.6 and 3.1 of Schöning.

Problems

0. Final project: everyone will have to do a final project for this course. This will involve investigating some topic related to logic and computer science – and reporting to me and the rest of the class via a short paper and presentation. This can be a somewhat applied project, i.e., learning how a tool works, i.e., the theory behind it, and putting it through its paces on an interesting problem; or reading a few papers about some topic.

Please think about what you would like to do and email me some questions and suggestions. On Tuesday I will bring to class a selected list of some of the many areas where logic is being applied to computer science, but I would be most happy if the topic that you end up with follows your interest. I would like to hear from all of you by the time this homework is due, with an idea or two about the kind of topics that you are thinking of investigating.

1. [10 pts.] Do Exercise 78, p. 87.
2. [15 pts.] Do Exercise 81, p. 89.
3. [15 pts.] Show using resolution that $\exists u\forall vP(v, u) \vdash \forall x\exists yP(x, y)$. That is, take the first formula and the negation of the second formula. Put them both in prenex form, Skolemize, and check that the quantifier-free parts are in CNF. Then use resolution to derive the empty clause.
4. [20 pts.] Do Exercise 85, p. 96.
5. [20 pts.] Do Exercise 86, p. 96.
6. [20 pts.]*
 - (a) Prove the lemma that we needed for the fundamental theorem of Ehrenfeucht-Fraïssé games, i.e., that if our vocabulary is finite and has no function symbols of arity greater than 0, then for each n there are only finitely many sentences up to equivalence of quantifier depth n . [Hint: prove this by induction, by giving an upper bound for how much this number can increase each time you increase the quantifier depth by one.]
 - (b) Give examples to show that the lemma is not true if there are either infinitely many relation symbols of arity one, or one function symbol of arity one.