CS601:	**Homework 11**	Due in main office before 4:30 pm, Tuesday, May 5, 2009

Problems:

1. [25 pts.] Prove that EMPTY-CFL, discussed in problem 7 of Hw2, is P complete. [Hint: I suggest you provide a reduction from MCVP to $\overline{\text{EMPTY-CFL}}$.]

2. [25 pts.] Show that NFA-$\Sigma^\star = \{N \mid N = (Q, \Sigma_N, \Delta, q_0, F)$ is an NFA; $\mathcal{L}(N) = \Sigma_N^\star\}$ is PSPACE complete.

   [Hint: For your reduction, construct an NFA that accepts all strings that do not encode a valid computation, similar to what we did in lecture 8.]

3. [25 pts.] As you can find in the slides for Lecture 21, the **polynomial-time hierarchy** (PH) consists of the set of problems in **alternating polynomial time** (ATIME$[n^{O(1)}]$) in which a bounded number of alternations are made between existential and universal states. In more detail,

   PH $= \bigcup_{i=1}^{\infty} \Sigma_k^p$, where Σ_k^p is the set of problems in ATIME$[n^{O(1)}]$ where the computation starts in an existential state and makes at most $k-1$ alternations between existential and universal states. Thus, $\Sigma_1^p =$ NP.

   Consider the problem Σ_k-QSAT which consists of true quantified boolean formulas of the form,

   $$\exists b_{11} \ldots b_{1c} \forall b_{21} \ldots b_{2c} \cdots Q_k b_{k1} \ldots b_{kc}(\alpha),$$

   where α is a boolean formula with boolean variables $b_{11} \ldots b_{kc}$. So, for example, Σ_1-QSAT is equivalent to SAT because a boolean formula, α, with boolean variables $x_1, \ldots, x_c$ is satisfiable iff the quantified boolean formula, $\exists x_1 \ldots x_c(\alpha)$ is true.

   Not surprisingly, Σ_k-QSAT is complete for Σ_k^p. Use this to prove that, if we assume that if we have a fixed bound, e.g., 3, on the arity of relations in our first-order vocabulary, then the problem, Σ_2-FO-SAT, which we studied in problem 1 of Hw8, is complete for Σ_2^p. Why do you need the assumption of bounded arity?

4. [25 pts.] Prove that NL $\subseteq$ sAC^1.

   [Hint: first show that REACH $\in$ sAC^1. It may help to at look slides 11 and 12 of Lecture 20. You should show that the family of circuits you produce is logspace uniform, i.e, the map $f : 1^n \mapsto C_n$ is computable in F(L).]