CS601:	**Homework 9**	Due in main office before 4:30 pm, Wednesday April 22, 2009

Problems:

1. [30 pts.] Define SO$\exists$-Horn to be the set of all formulas,

$$\Phi \;\equiv\; \exists R_1^{a_1} \cdots R_k^{a_k} \forall x_1 \cdots x_r(\psi),$$

such that $\Phi \in$ SO$\exists$ and the first-order part of Φ is universal and the quantifier-free part, ψ, is a Horn formula in the sense that it is in CNF and each clause has at most one positive literal such as $R_i(v_1, \ldots, v_{a_i})$. That is we don't care about positive literals involving the numeric relations, i.e., $=, \leq,$ Suc, or input relations, e.g., E, only about the existentially quantified relations R_i.

For example, the clause, $(R_i(x_1) \vee \neg R_j(x_2) \vee E(x_1, x_2) \vee x_1 = x_2)$ counts as Horn because there is only one positive literal, $R_i(x_1)$ that counts. $E(x_1, x_2)$ does not count because it uses an input relation and $x_1 = x_2$ does not count because it uses a numeric relation. However, the clause $(R_i(x_1) \vee R_j(x_2))$ is not Horn.

(See Papadimitriou, or the slides from lecture 10 for an algorithm showing that HORN-SAT $\in$ P.)

   (a) Prove that SO$\exists$-Horn $\subseteq$ P.

   (b) Modify the argument in my proof of Fagin's Theorem from Lecture 18 to prove Grädel's Theorem: SO$\exists$-Horn $=$ P. Note: we are assuming that all structures are given the numerical relations $\leq,$ Suc and constants $0, \max$.

   (c) Modify the argument in my proof of Cook's Theorem from Fagin's Theorem from Lecture 18 to prove, using Grädel's Theorem, that HORN-SAT is P-complete.

2. [20 pts.] Prove that EMPTY-DFA, the language of all DFAs D such that $\mathcal{L}(D) = \emptyset$, is NL complete.

3. [20 pts.] Prove that the following problem is P complete:

$$U_{\text{time}} \quad = \quad \{x\#w\#^r \mid M_x \text{ accepts } w \text{ in less than or equal to } r \text{ steps}\}$$

4. [30 pts.] Here is a pair of related problems concerning the complexity class Primitive Recursive.

   (a) Let $t : \mathbf{N} \to \mathbf{Z}^+$ be any primitive recursive function. Prove using the COMP Theorem that

$$\text{DTIME}[t(n)] \subseteq \text{Primitive Recursive}$$

   [To do this you should look somewhat carefully at the encoding in the COMP theorem and at the definition of DTIME$[t(n)]$.]

   Conclude that P $\subseteq$ Primitive Recursive, EXPTIME $\subseteq$ Primitive Recursive, DTIME[hyperExp(n)] $\subseteq$ Primitive Recursive, etc.

   (b) Show that there is no problem that is complete for Primitive Recursive, via logspace reductions.