Notes and Comments: Today we proved the completeness and compactness theorems for propositional logic. The completeness theorem shows that proofs in propositional logic ($\Gamma \vdash \varphi$) is the same thing as semantic implication ($\Gamma \models \varphi$) and thus makes a basic and important connection between finite derivations and truth. The compactness theorem is a corollary: sinces proofs are finite, the notion of consistency is finite and thus by the completeness theorem so is the notion of satisfiability.

Now we have completed the unit on propositional logic. Today we also started Predicate Logic, making the basic and important definitions of **structure**, **vocabulary** (similarity type), **TERM**, and **FORM**, the latter being the set of syntactic objects of predicate logic.

Reading: Please read pages 55 through 89 of van Dalen by Sept. 25.

Example: While Propositional Logic is too simple a model, Predicate logic is a rather good model of mathematics. As an example, I started to show you that in FORM, i.e., in predicate logic, we can write much of what we would want to say about the real numbers. Consider the structure $\mathcal{A}_f = \langle \mathbf{R}, +, \cdot, f, 0, 1\rangle$ and the corresponding vocabulary, where we will write $+$ and $\cdot$ in infix notation, and we will let f denote some arbitrary unary function $f : \mathbf{R} \to \mathbf{R}$. We can write many interesting things in FORM, but it is nice to make some abbreviations as we go.

$$\begin{aligned}
x \neq y &:= (\neg x = y) \\
x^2 &:= (x \cdot x) \\
x < y &:= \exists z(z \neq 0 \ \wedge \ x + z^2 = y) \\
|x - y| < \delta &:= \exists z(z + y = x \ \wedge \ z^2 < \delta^2) \\
(\forall \epsilon > 0)(\varphi) &:= \forall \epsilon(\epsilon > 0 \to \varphi) \\
(\exists \delta > 0)(\varphi) &:= \exists \delta(\delta > 0 \ \wedge \ \varphi) \\
\lim_{x \to y}(f(x)) = w &:= (\forall \epsilon > 0)(\exists \delta > 0)\forall z(|y - z| < \delta \ \to \ |w - f(z)| < \epsilon) \\
\text{“}f \text{ is continuous”} &:= \forall y(\lim_{x \to y}(f(x)) = f(y))
\end{aligned}$$