Notes and Comments: In the first lecture, Sept. 4, 1996, I gave an overview of what this course is about as described on the course's homepage. Next, I gave an introduction to logic and I started Propositional Logic as in van Dalen, pp. 5 – 9.

Reading: Please read van Dalen, pp. 1 – 20. Please try all the exercises as you come to them. In particular, the exercises on pp. 13 – 14, and 19 – 20 are all good for you to do. I mean that you should think about them and try them and see how to do them. You do not need to write them up. If you have questions about the exercises or the reading please ask. You may ask a friend, colleague, or me, especially in class, in my office hours, or by email. In general, I think of exercises as to be thought about by you and not needing handing in. I will give a few problem sets which are slightly more formal and interesting than exercises. The more you are diligent about trying the exercises and making sure that with help you can do them all, the less I will need to give problem sets.

Another Exercise: Here are some further comments and an exercise concerning the definition of PROP, page 7 of van Dalen. Recall that the alphabet for propositional logic is

$$\Sigma \quad = \quad \{p_i \mid i \in \mathbf{N}\} \cup \{\bot, \wedge, \vee, \rightarrow, \leftrightarrow, \neg, (,)\}$$

Recall that PROP is defined as **the smallest subset** $X \subseteq \Sigma^\star$ satisfying the following three properties:

(i) $p_i \in X$ for all $i \in \mathbf{N}$, and $\bot \in X$.

(ii) If $\varphi, \psi \in X$ then $(\varphi \Box \psi) \in X$ for $\Box \in \{\wedge, \vee, \rightarrow, \leftrightarrow\}$

(iii) If $\varphi \in X$ then $(\neg\varphi) \in X$.

But, what do we mean by, "**the smallest subset** $X \subseteq \Sigma^\star$ satisfying (i), (ii), and (iii),"?

Answer: what this means is,

$$\text{PROP} \quad = \quad \bigcap\{X \subseteq \Sigma^\star \mid X \text{ satisfies (i), (ii), (iii)}\}$$

To make sense of this definition you should do the following:

1. Show that there is at least one set $X \subseteq \Sigma^\star$ satisfying (i), (ii), (iii), so the intersection is not vacuous. Hint: $X = \Sigma^\star$ works.
2. Show that PROP as defined by this intersection satisfies (i), (ii), and (iii).
3. Show that if $X \subseteq \Sigma^\star$ and X satisfies (i), (ii), and (iii), then PROP $\subseteq X$.