Comp Sci 691M:	Second Problem Set	Due: Oct. 16, 1996

Please do the following problems:

1. Do exercise 13 on page 117 of van Dalen, i.e., prove that if T_1 and T_2 are theories in the same language, but have no models in common, then there is a sentence σ such that $T_1 \vdash \sigma$ and $T_2 \vdash \neg\sigma$.

2. Do exercise 18 on page 118 of van Dalen: a graph is k-colorable iff every finite subgraph is k-colorable.

3. Do exercise 3 on page 132 of van Dalen: preservation theorems for universal and for existential formulas.

4. A dense linear ordering without endpoints (DLOWE) is a model of the following axioms:

   reflexive: $(\forall x)R(x,x)$
   antisymmetric: $(\forall xy)((R(x,y) \wedge R(y,x)) \rightarrow x \equiv y)$
   transitive: $(\forall xyz)((R(x,y) \wedge R(y,z)) \rightarrow R(x,z))$
   total: $(\forall xy)(R(x,y) \vee R(y,x))$
   dense: $(\forall xy)((R(x,y) \wedge x \not\equiv y) \rightarrow (\exists z)(R(x,z) \wedge R(z,y) \wedge x \not\equiv z \wedge z \not\equiv y))$
   wo endpoints: $(\forall x)(\exists yz)(R(y,x) \wedge R(x,z) \wedge x \not\equiv y \wedge x \not\equiv z))$

   Let $\mathcal{A}$ and $\mathcal{B}$ be any two countable models of DLOWE. Prove that $\mathcal{A} \cong \mathcal{B}$. (A theory with this property is called $\aleph_0$-categorical.

5. Let $\mathbf{N}$ be the structure consisting of the natural numbers with R interpreted as $\leq$. Let $\mathbf{N}+\mathbf{Z}$ be the disjoint union of the naturals and the integers where R is $\leq$ on $\mathbf{N}$ and $\mathbf{Z}$, and $R(n,z)$ holds whenver $n \in \mathbf{N}$ and $z \in \mathbf{Z}$.

   Prove using Ehrenfeucht-Fraissé games that $\mathbf{N}$ is elementarily equivalent to $\mathbf{N}+\mathbf{Z}$. You should also show that the two structures are not isomorphic.