THE THROUGHPUT ORDER OF AD HOC NETWORKS EMPLOYING NETWORK CODING AND BROADCASTING

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Abstract— **Gupta and Kumar established that the per node throughput of ad hoc networks with multi-pair unicast traffic scales as $\lambda(n) = \Theta(1/\sqrt{n \log n})$, thus indicating that network performance does not scale well with an increasing number of nodes. However, the model of Gupta and Kumar did not allow for the possibility of network coding and broadcasting, and recent work has suggested that such techniques have the potential to greatly improve network throughput. Here, for multiple unicast flows in a random topology under the protocol communication model of Gupta and Kumar [1], we show that for arbitrary network coding and broadcasting in a two-dimensional random topology that the throughput scales as $\lambda(n) = \Theta(1/nr(n))$ where n is the total number of nodes and $r(n)$ is the transmission radius. When $r(n)$ is set to ensure connectivity, $\lambda(n) = \Theta(1/\sqrt{n \log n})$, which is of the same order as the lower bound for the throughput without network coding and broadcasting; in other words, network coding and broadcasting at most provides a constant factor improvement in the throughput. This result is also extended to one-dimensional and three-dimensional random deployment topologies, where it is shown that $\lambda(n) = \Theta(1/n)$ for the one-dimensional topology and $\lambda(n) = \Theta(\frac{1}{\sqrt[3]{n \log^2 n}})$ for three-dimensional networks.**

I. INTRODUCTION

Multi-hop wireless mesh networks have been intensively studied in recent years for both commerical and government applications. Such networks have the potential to serve as either a self-contained network that provides communication without the presence of an established infrastructure, or as a ubiquitous bridge between end users and the high speed wired infrastructure. Potentially such wireless mesh networks can be deployed in the streets of big cities, campuses, conference centers, combat fields, etc. Hence, issues of the connectivity and capacity of this type of networks are of interest.

One major concern with such wireless networks is scalability. In particular, under a traditional communication model, Gupta&Kumar [1] shows that the per node throughput of such random networks scales as $\lambda(n) = \Theta(\frac{1}{\sqrt{n \log n}})$ where n is the total number of nodes in the network and each node needs to send to a randomly chosen destination node with a rate of $\lambda(n)$. This result shows that as the total number of nodes increases, the many to many throughput decreases polynomially. However, recent work by Ahlswede, Cai, Li and Yeoung [2] introduces the concept of network coding (NC), and there has been tremendous interest in applying network coding in both wired [3] and wireless networks [4] [5] [6]. In the case of wireless multi-hop networks, applying network coding can potentially improve the performance on throughput [5] [6], energy efficiency and congestion control [4] [5]. In addition, recent work by Katti et al [7] demonstrates the potential throughput benefit of applying network coding to wireless networks through constructive examples and experiments. Since network coding was not taken into consideration in Gupta&Kumar's original work [1] and the related works that followed, an interesting question raised after [7] is how much throughput benefit can it provide to such networks. Answering this question will help us to better understand not only the benefit and limitations of network coding in wireless networks but also the degree of scalability of such random wireless networks.

The idea of [7] is to broadcast combined information (coded) of intersecting flows, and then each flow's next hop relay node is able to decode its relaying flow's traffic based on all of the broadcasts that it has received as well as on local information (source data generated locally). In this way a node can potentially deliver to multiple neighboring nodes multiple data flows with a single broadcast transmission. An example of this is shown in Fig. 1. Without network coding and broadcasting, four transmissions are required; however, when the opportunistic algorithm of [7] is applied, with the middle node broadcasting the XOR of a and b, only three transmissions are required to move packets a and b forward two hops. Thus, intuitively it appears that there could be a throughput benefit ratio proportional to the expected number of neighbors $\Theta(\log n)$. However, here we demonstrate that such a large improvement is not possible; in fact, only a constant improvement in throughput can be achieved. At first, this result might seem counter-intuitive, since, with network coding and broadcasting, each node can send information to all neighbors with one transmission. However, essentially each node still needs to receive information one transmission at a time. In other words, whereas there is simultaneous transmission, there is not simultaneous reception. Since the information flow rate across any node also needs to be conserved, the incoming information rate will be a bottleneck for the throughput improvement.

More formally, first we derive an upper bound on the throughput of schemes aided by network coding and broadcasting. We do this by analyzing the information rate across a sparsity cut of the network. From geometric constraints on the receivers' locations we derive an upper bound on the maximum number of simultaneous transmissions across the sparsity cut. This, combined with a coding constraint, yields the throughput upper bound. After obtaining this upper bound for coding schemes, we show that non-coding schemes achieve the same order throughput, and thus network coding can provide at most a constant factor improvement on the many to many throughput. These results are shown for random networks deployed in $1D, 2D, 3D$ and in general kD Euclidean space.

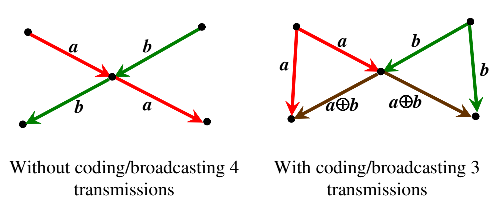

Fig. 1. An example demonstrating the benefit of Katti etc. [7]'s opportunistic coding scheme

II. MODEL FORMULATION

We consider the network model of Gupta&Kumar [1], where n nodes are randomly located, i.e., independently and uniformly distributed, in a region of fixed area. Gupta&Kumar [1] study two types of regions: a disk and the surface of a sphere, both with an area of one meter squared. More generally, we do not limit the shape of the region or its dimension. However, for simplicity of presentation, we derive our results based on a unit square in two dimensions ($2D$), a unit line segment in one dimension ($1D$), and a unit cube in three dimensions ($3D$).

There are n source-destination pairs in the network. Each node i in the network is a data source that needs to route its data through multi-hop wireless communications to a destination node that is independently and uniformly randomly chosen. The same protocol and physical communication models as in Gupta&Kumar [1] are employed. For the protocol model, as shown in Fig. 2, a transmission from node i to j is successful iff the distance between them satisfies $|X_i - X_j| \leq r(n)$ and any other simultaneously transmitting node k satisfies $|X_k - X_j| \geq (1+\Delta)r(n)$. Here, X_i is node i's location, $r(n)$ is the transmission radius and $\Delta > 0$ ensures a safety zone that limits the interference; in particular, Δ is a constant that depends on the properties of the wireless medium. In addition, there is a finite bandwidth limit of W bits/sec for each transmission. In order to ensure connectivity, the fixed transmission radius for the protocol model needs to be at least $r(n) = \Theta(\frac{\sqrt{\log n}}{\sqrt{n}})$ [1].

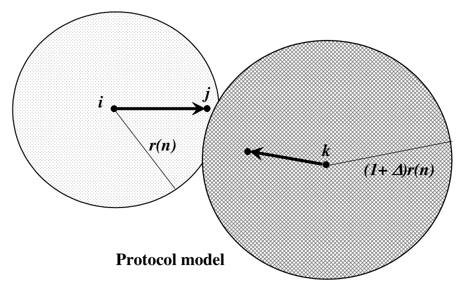

Fig. 2. The protocol communication model

As in Gupta&Kumar [1], attention here is focused on the many to many throughput of the network, i.e. the data rate at which each node can send to its destination node. A throughput $\lambda(n)$ (bits/sec) is *feasible* if there exists a scheme that achieves $\lambda(n)$ on average. The *throughput capacity* of such a random network is defined as the maximum throughput that is feasible with high probability.

Here, transmission schemes correspond to same type of "spatial and temporal scheduling schemes that operate the network in a multi-hop fashion and buffers at intermediate nodes when awaiting transmissions" as in [1]. Two types of schemes are considered: a *flow scheme* and a *coding scheme*. A flow scheme is a non-coding scheme where data are routed as commodity flows (duplication, forwarding, but no coding) and thus the broadcast nature of the wireless medium is not helpful for our unicast task. Gupta&Kumar[1] studied the throughput of flow schemes. A coding scheme is one that allows all of the operations in a flow scheme, along with allowing messages received at each node to be decoded/recoded; in other words, intermediate nodes can send the results obtained from applying arbitrary functions to all previously received bits and its own source data as long as each destination node is able to decode the data intended for it from all of its received bits and local data. Thus all possible benefits of combining network coding and wireless broadcasting as demonstrated in Katti etc. [7] are incorporated in the considered coding schemes. The throughput capacity is denoted as $\lambda_F(n)$ for flow schemes and $\lambda_C(n)$ for coding schemes. The *throughput benefit ratio* of the coding scheme is denoted $\alpha(n) = \frac{\lambda_F(n)}{\lambda_C(n)}$. As in Gupta&Kumar [1], all packets are independent of each other whether they are from different sources or the same source; in other words, there are no spatial or temporal correlations among the source data.

III. Throughput Order of Coding Scheme

In this section, the main results are presented. In particular, we show that coding schemes provide at most a constant factor improvement in throughput over flow schemes. In other words, there exists some constant c (i.e. not dependent on n) such that $\alpha(n) \leq c$.

A. 2D case

The main result is proved for the $2D$ case first. First, the *sparsity cut* of a random network is defined, and an upper bound on the maximum number of simultaneous transmissions across any cut is derived in Lemma 1.

In general, a cut Γ is defined as a partition of the nodes in a graph, the cut capacity is the sum of the links' bandwidths crossing the cut, and the sparsity cut is a cut where the cut capacity divided by the traffic demand is the minimum over all cuts. Since the network studied here is a random network embedded in an Euclidean space and transmissions are between neighboring nodes, attention can be focused on a narrow class of cuts that are induced by a line segment (or a plane in the $3D$ case) that cuts the region into two regions. The *cut length* l_Γ is defined as the length of the cut line segment. Denote the two subregions divided by the cut as Γ_1 and Γ_2. A *sparsity cut* for a random network is defined as a cut induced by the line segment with the minimum length that separates the region into two equal area subregions. For the square deployment region illustrated in Fig. 3, the line segment AB induces a sparsity cut Γ_{AB}. Since nodes are uniformly randomly deployed in a random network, such a sparsity cut captures the traffic bottleneck of such a random network on average. The *cut capacity* is defined as $(\Lambda_{\Gamma_{1,2}}, \Lambda_{\Gamma_{2,1}})$ where $\Lambda_{\Gamma_{1,2}}$ equals the transmission bandwidth W times the maximum possible number of simultaneous transmissions (broadcast or non-broadcast) across the cut from Γ_1 to Γ_2 [1]; and $\Lambda_{\Gamma_{2,1}}$ equals the same quantity from Γ_2 to Γ_1. This cut capacity constrains the information rate that the nodes from one side of the cut as a whole can deliver to the nodes at the other side as a whole. The number of sources in Γ_1 whose destinations are in Γ_2 is denoted as $n_{\Gamma_{1,2}}$. The cut capacity is bounded by deriving an upper bound

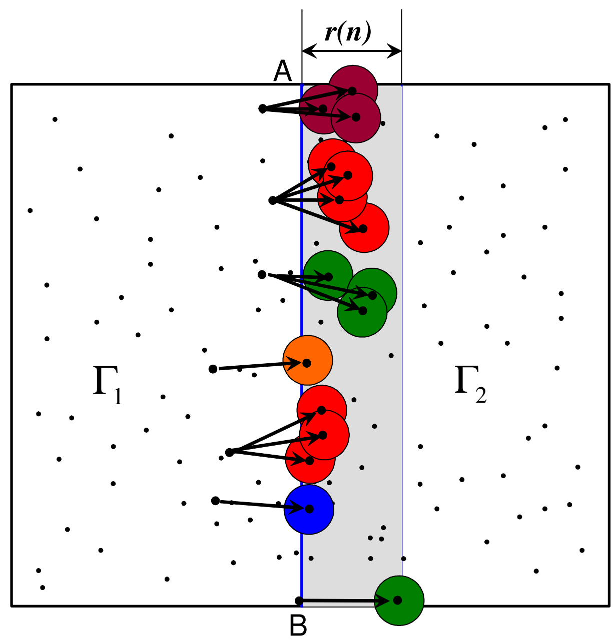

Fig. 3. Cut Capacity in 2D

on the maximum number of simultaneous transmissions across the cut. It is easy to see that all of the direct receivers of transmissions across a cut Γ in one direction lie in the shaded rectangle region with area $l_\Gamma \times r(n)$ as shown in Fig. 3. In [1], disks of radius $\frac{\Delta r(n)}{2}$ centered at each receiver are disjoint[2]. However, [1] does not exploit broadcast transmissions while a coding scheme does. As shown in Fig. 4, with the consideration of broadcast

[1]The maximum number of bits of information per second that can be transmitted across the cut from one side (Γ_1) to the other (Γ_2)

[2]Otherwise some sender is within $(1 + \Delta)r(n)$ of some other sender's receiver.

and network coding, observe that such disks centered at receivers of the same sender (broadcast transmission) could overlap, disks centered at receivers of different senders are still disjoint. In other words, we have the following Observation:

Observation 1: The union of disks (with radius $\frac{r(n)}{2}$) centered at the receivers of one transmission should be disjoint from the union of disks centered at the receivers of another transmission.

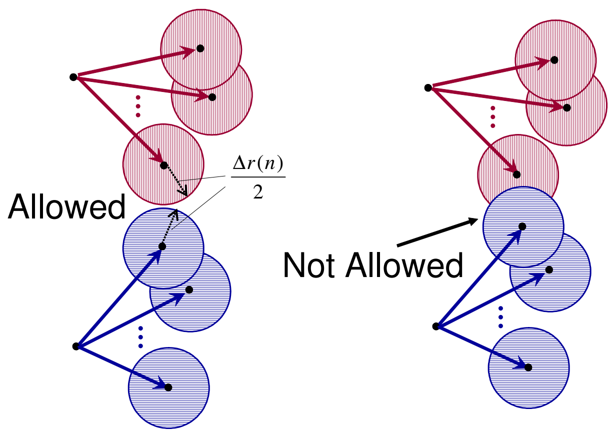

Fig. 4. Interference of coding schemes in 2D

Lemma 1: The capacity of a cut Γ *for a* $2D$ *region has an upper bound of* $\frac{c_\Delta l_\Gamma W}{r(n)}$ *where* $c_\Delta = \max\{\frac{16}{\pi\Delta^2}, \frac{\sqrt{3}}{\Delta}\}$

Proof: When $\Delta < 2$, Observation 1 means each transmission across the cut consumes at least an area of $\frac{1}{4}\pi(\frac{\Delta r(n)}{2})^2$ of the shaded region in Fig.3, with this minimum achieved when receiver lies in the corner of the shaded region. Thus, the maximum number of simultaneous transmissions across the cut is upper bounded by the area of the shaded region divided by $\frac{1}{4}\pi(\frac{\Delta r(n)}{2})^2$, which is $\frac{16 l_\Gamma}{\pi\Delta^2 r(n)}$. When $\Delta \geq 2$, as shown in Fig. 5, any two receivers of two different transmissions require a $\frac{\sqrt{3}}{2}\Delta r(n)$ difference in their coordinates along the cut line. Thus, there can be at most $\frac{l_\Gamma}{\sqrt{3}\Delta r(n)/2} + 1 \leq \frac{\sqrt{3} l_\Gamma}{\Delta r(n)}$ simultaneous transmissions across the cut.

Since each transmission is able to send W bits/sec, combining the two cases above, the cut capacity is upper bounded by $\Lambda_{\Gamma_{1,2}} \leq \frac{c_\Delta l_\Gamma W}{r(n)}$ and $\Lambda_{\Gamma_{2,1}} \leq \frac{c_\Delta l_\Gamma W}{r(n)}$, where $c_\Delta = \max\{\frac{16}{\pi\Delta^2}, \frac{\sqrt{3}}{\Delta}\}$. ■

Corollary 1: The sparsity cut capacity of a $2D$ *random network has an upper bound of* $\frac{c_\Delta W}{r(n)}$.

Proof: Regardless of the shape of the unit area region, there exists a sparsity cut for each orientation of

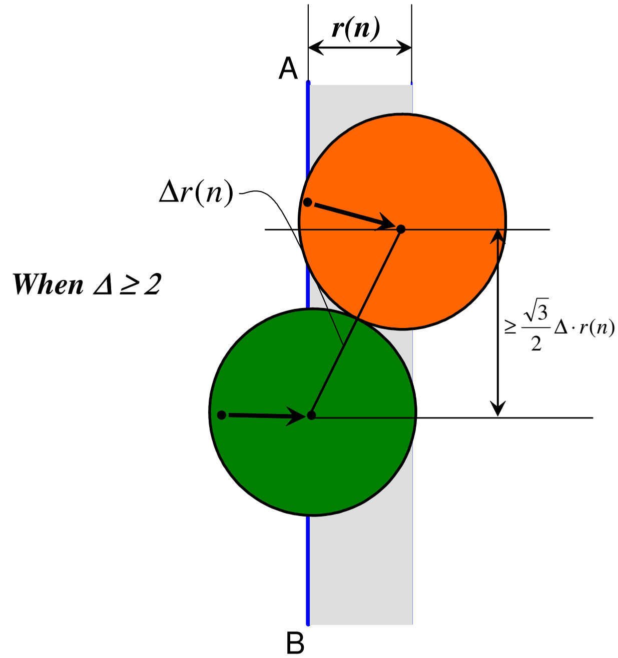

Fig. 5. Cut capacity $\Delta \geq 2$ case

the cut line. If we rotate the cut line, there has to be at least one sparsity cut with cut length $l_\Gamma \leq 1$. Hence, from Lemma 1, the corollary is proved. ■

Next an upper bound for the throughput of coding schemes in a $2D$ random network is derived.

Theorem 1: The throughput of coding schemes in a $2D$ *random network is upper bounded by* $\Theta(\frac{W}{nr(n)}) = \Theta(\frac{W}{\sqrt{n \log n}})$

Proof: Assume the coding throughput of the n node random network is $\lambda_C(n)$. Then, by its definition, with high probability (w.h.p.) there exists some coding scheme that, for some $T < \infty$, during each time interval $[(i-1)T, iT]$ (in seconds), every node can send $T\lambda_C(n)$ bits of information to its corresponding destination node. For a sparsity cut Γ_{AB} in the middle, by a Chernoff bound [8] argument it is easy to see that w.h.p. there are $\Theta(n)$ pairs of source-destination nodes that need to cross Γ_{AB} in one direction, or $n_{\Gamma_{1,2}} = n_{\Gamma_{2,1}} = \Theta(n)$, w.h.p.. Now we view all of the nodes lying on the right side of AB as a super node, and treat all of the distinct messages it receives from the left side of AB within the time interval $[(i-1)T, iT]$ as a single 'meta' message M. We denote the number of bits of M as B_M. According to the definition of our coding scheme, this meta message M can be arbitrarily coded but with only one coding constraint: by Shannon's data compression theorem [9], in order for the right side destination nodes to decode the original data from the left side sources which are independent of each other, M has to satisfy $B_M \geq T n_{r_{1,2}} \lambda_C(n)$ or $B_M \geq T\Theta(n)\lambda_C(n)$ w.h.p..

At the same time, our work above has provided a capacity constraint that upper bounds B_M. A broadcast transmission across the cut to multiple receivers delivers identical information to the receivers; however, by the definition of M, the identical messages will only be counted once in M for one of the receivers. Also by the definition of cut capacity and Corollary 1, we get $B_M \leq \frac{c_\Delta W}{r(n)} T$. Combined with the coding constraint above, we derive $\lambda_C(n) \leq \frac{c_\Delta W}{\Theta(n) r(n)}$ w.h.p.. Since $r(n)$ is at least $\Theta(\frac{\sqrt{\log n}}{\sqrt{n}})$ to ensure connectivity [1], and the definition of throughput is already a high probability quantity, we have $\lambda_C(n) \leq \frac{c_\Delta W}{\Theta(\sqrt{n \log n})}$. ■

Theorem 2: The $2D$ throughput benefit ratio is upper bounded by a constant:

$$\alpha(n) = \Theta(1)$$

Proof: Gupta&Kumar [1] already establishes a lower bound for the throughput of flow schemes, $\lambda_F(n) \geq \Theta(\frac{c_1 W}{(1+\Delta)^2 \sqrt{n \log n}})$ where $c_1 > 0$ is a constant. Combined with Theorem 1, we get $\alpha(n) = \frac{\lambda_C(n)}{\lambda_F(n)} = \Theta(1)$. ■

This constant throughput benefit ratio is true for a random network deployed in any arbitrarily shaped region. First, the upper bound for the throughput of the coding scheme still holds. Second, the constructive lower bound of Gupta&Kumar [1] can in fact be extended to arbitrarily shaped regions, even though the asymmetry may cause the constructive scheme in [1] to have a skewed load distribution for some cut. Since the region is of fixed area, the throughput loss due to the asymmetric shape will be a fixed constant factor as n increases. Thus, we have shown that network coding combined with wireless broadcast provides no order-different improvement on the throughput of a random network deployed in any arbitrarily shaped $2D$ region.

B. 1D case

The $1D$ case is easier to deal with than the $2D$ case. Traffic either goes left or right along one line. However, $1D$ differs from $2D$ in that the transmission radius does not affect the order of throughput. Thus, we are able to give a constructive lower bound for the $1D$ throughput that does not have a $\log n$ factor as in the $2D$ and $3D$ cases.

First we make the following straightforward observation.

Observation 2: For any cut in the $1D$ line, there can be at most one transmission across the cut (including both directions) at any given point in time.

Lemma 2: The throughput of the coding scheme on a $1D$ random network is upper bounded by

$$\lambda_C(n) \leq \frac{2W}{n}$$

Proof: We prove it by showing that for any given constant $\epsilon > 0$ that is arbitrarily small, $\lambda_C(n) \leq \frac{2W}{(1-\epsilon)n}$ for large n.

We consider the sparsity cut Γ_m that cuts the line segment in the middle. Using a Chernoff bound [8], it is easy to show that the number of sources that need to send data across Γ_m from left to right is larger than $(1-\epsilon)\frac{n}{4}$ w.h.p., and that the same is true for the number of sources crossing the cut from right to left.

Applying the same technique as in Theorem 1 and by Observation 2, we have $\lambda_C(n) 2(1-\epsilon)\frac{n}{4} \leq W$, which yields the desired result. ■

To our knowledge, there is no constructive lower bound for the throughput of a $1D$ random network yet. In this paper, we first construct a lower bound for a $1D$ random network and show that its throughput capacity is on the order of $\Theta(\frac{W}{n})$.

Lemma 3: The throughput of flow schemes on a $1D$ random network is lower bounded by

$$\lambda_F(n) \geq \frac{c_{\Delta_2} W}{n}$$

where $c_{\Delta_2} = \min\{\frac{1}{\lceil 2\Delta \rceil + 2.75}, \frac{1}{4}\}$.

Proof: We prove it by showing that for any given constant $\epsilon > 0$ that is arbitrarily small, $\lambda_F(n) \geq \frac{c_{\Delta_2} W}{(1+\epsilon)n}$ for large n.

We choose a transmission radius $r(n) = \frac{40 \log n}{n}$, divide the line deployed region into bins each of length $\frac{r(n)}{2} = \frac{20 \log n}{n}$ and all together $\frac{n}{20 \log n}$ bins. Then by the same union bound argument as in [10], w.h.p. every bin contains at least one node. Furthermore, a node in one bin can reach any node in an adjacent bin directly.

Routing consists of hopping from one bin to the next bin in direction of the destination unless the destination is in the same bin as the source. Any node in the next bin can be a relay but for the last bin the algorithm will choose the destination node itself.

Next we show that there exists a temporal scheduling scheme that on average allows each bin a chance to transmit W bits to each of its two neighboring bins every $4/c_{\Delta_2}$ seconds. This is done by mapping the bin-hopping transmissions to a new graph. For each bin we construct two virtual vertices in a new graph, one for transmissions from this bin going to the right, one for transmissions going left. We connect any two vertices in the new graph with an edge if the transmissions that they represent could potentially interfere with each other when occurring simultaneously. When $\Delta < 1$, each vertex in the new graph will have a degree of at most 15. By the graph coloring theorem [11], $15+1$ colors are enough to color the vertices s.t. no two interfering vertices (transmissions) have the same color. This gives a schedule of length 16 where each bin gets a chance to transmit to both directions. When $\Delta \geq 1$, we count the potential interfering bin transmissions and it is not hard to see that the vertex degree is at most $4\lceil 2\Delta \rceil + 10$, so there is a schedule of length $4\lceil 2\Delta \rceil + 11$. Combining these two cases, we have on average that every $4/c_{\Delta_2}$ seconds each bin will get a chance to transmit one second (W bits) for both directions.

There are altogether $K = \frac{n}{20 \log n}$ bins. Denote the sum of the number of source and destination nodes in each bin as $b_1, b_2, \ldots, b_K$. Then, using the Chernoff bound and the union bound of probability, we have $b_i < \frac{50 \log n}{n}$ for all $i = 1, \ldots, K$ simultaneously w.h.p.. Also, using the Chernoff bound and union bound, for any cut, the number of sources that need to send traffic across the cut is upper bounded by $(1+\epsilon)\frac{n}{4}$ for each of the two directions, where $\epsilon > 0$ is an arbitrarily small constant. Thus a throughput of $\lambda(n) = \frac{W}{4/c_{\Delta_2}((1+\epsilon)\frac{n}{4} + \frac{50 \log n}{n})}$ is achievable w.h.p., because there exists a schedule that can deliver $\lambda(n)(1+\epsilon)\frac{n}{4}$ bits/sec across any cut w.h.p. This throughput is just $\lambda(n) = \frac{c_{\Delta_2} W}{((1+\epsilon)n + \frac{200 \log n}{n})} = \frac{c_{\Delta_2} W}{(1+\epsilon+\frac{200 \log n}{n^2})n}$. Since $\frac{200 \log n}{n^2} \to 0$ as $n \to \infty$, it can be absorbed into the ϵ term, yielding the lemma. ∎

Theorem 3: The $1D$ throughput improvement of the coding scheme over the flow scheme is at most a constant factor; more specifically:

$$\alpha(n) \leq \frac{2}{c_{\Delta_2}}$$

where where $c_{\Delta_2} = \min\{\frac{1}{\lceil 2\Delta \rceil + 2.75}, \frac{1}{4}\}$.

Proof: This follows directly from Lemmas 2 and 3. ∎

From Theorem 3, we see that, as in $2D$, coding schemes provide no order different throughput improvement in $1D$. One thing to notice is that in $1D$ there is no $\log n$ factor in the throughput. The reason that such can be achieved is because in $1D$ the transmission radius has no order difference effect on the throughput. Intuitively, in $2D$, as we increase the transmission radius $r(n)$, the number of hops and the relaying traffic for each node drops linearly, while the spatial multiplexing drops quadratically, and thus the joint effect on throughput will be decreasing linearly on $r(n)$. However, for $1D$, as we increase $r(n)$, the spatial multiplexing also drops linearly; thus, there is no order difference effect on throughput no matter what $r(n)$ we choose, and the $\log n$ factor can be eliminated.

In [12], percolation theory is employed to construct a lower bound of the throughput in $2D$ that removes the $\log n$ factor. Essentially, this is achieved by using smaller $r(n)$ for the majority of the transmissions. For $1D$, the percolation result does not hold, but our result shows that the percolation technique is not required to remove the $\log n$ factor.

C. 3D case

The $3D$ case is similar to the $2D$ situation. Applying the same technique as in $2D$, similar results follows as below.

Theorem 4: The throughput of the coding scheme on a $3D$ random network is upper bounded by $\lambda_C(n) \leq \Theta(\frac{W}{\sqrt[3]{n \log^2 n}})$. More specifically, $\lambda_C(n) \leq \frac{c_{\Delta_3} W}{n r(n)^2}$ where $c_{\Delta_3} = \min\{\frac{192}{(1-\epsilon)\pi\Delta^3}, \frac{256}{\sqrt{3}(1-\epsilon)\pi\Delta^2}\}$ and $\epsilon > 0$ is an arbitrarily small constant.

Proof: The proof uses the same technique as in the $2D$ and $1D$ cases. One difference is now that, when $\Delta < 2$, each transmission occupies at least a volume of $\frac{4}{3}\pi(\frac{\Delta r(n)}{2})^3 \frac{1}{8}$, i.e. one eighth of a sphere of radius $\frac{\Delta r(n)}{2}$; when $\Delta \geq 2$, each transmission will occupy at least one fourth of a circle with radius $\frac{\sqrt{3}\Delta r(n)}{4}$ on the cut plane. Another difference is that ensuring connectivity in $3D$ requires that $r(n) = \Theta(\sqrt[3]{\frac{\log n}{n}})$. The rest of the argument is the same. ∎

Theorem 5: The $3D$ throughput benefit ratio is upper bounded by a constant:

$$\alpha(n) = \Theta(1)$$

Proof: Gupta&Kumar have already shown a constructive lower bound for the throughput of flow schemes in [13], $\lambda_F(n) \geq \Theta(\frac{W}{(n \log^2 n)^{\frac{1}{3}}})$. Combined with Theorem 4, we get $\alpha(n) = \Theta(1)$. ∎

In general, we can show that for a random network in an abstract kD ($k > 1$) Euclidean space, the coding scheme provides no order different benefit on throughput. More specifically, the coding throughput is upper bounded by $\lambda_C(n) \leq \Theta(\frac{W}{\sqrt[k]{n \log^{k-1} n}})$, and there exists a flow scheme achieving the same order throughput.

IV. Conclusion and Future Work

In this work, we have studied the potential benefit of network coding & broadcasting on the many to many throughput of wireless networks under the framework proposed by Gupta and Kumar [1]. In particular, we have shown that the benefit is upper bounded by a constant.

The future work includes deriving tight bounds on the constant value of the throughput benefit ratios, studying possibilities of improving the throughput with other forms of coordination among the wireless nodes, and studying the impact of network coding on delay and buffer size.

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