CMPSCI 240: Reasoning Under Uncertainty
Discussion 8

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Bayesian Network Graph: Consider the following scenario: The grass in my backyard is wet if it rains or if the sprinkler is on. If it is a cloudy day, then it is more likely that it will rain. I'm also trying to save water, so I usually don't turn the sprinkler on on cloudy days.

What random variables can we use to describe this problem? What are the dependencies between them? Draw a graph.

Bayesian Network - Graph to Factors: What is the factorization associated with the Bayesian network graph for the above example?

Bayesian Network - Factors to Graph: Suppose we have a Bayesian network with the following factorization. Draw the corresponding Bayesian network graph.

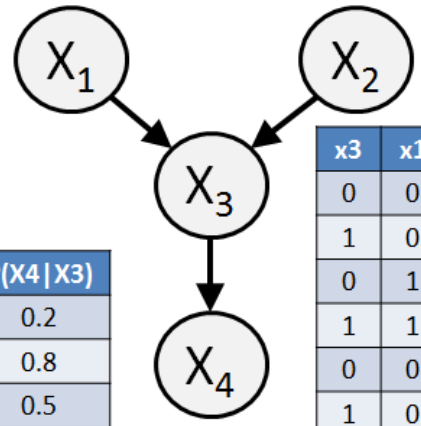
$$P(X_1, X_2, X_3, X_4) = P(X_1|X_2, X_3)P(X_2)P(X_3|X_2)P(X_4|X_1, X_2, X_3)$$

Bayesian Network Storage: If all the nodes are binary, what is the minimum space needed to store the parameters of the Bayesian network shown above?

Bayesian Network Storage: What is the minimum space needed to store the parameters of the Bayesian network shown above, if X_1 has 10 states, and every other node is binary?

Bayesian Network Queries: Use the Bayesian network shown below to compute the following probability queries: $P(X_1 = 1, X_2 = 0, X_3 = 0, x_4 = 1)$, $P(X_1 = 1, X_3 = 1)$ and $P(X_1 = 1 | X_2 = 0, X_3 = 0, X_4 = 1)$.

x1	P(X1)
0	0.4
1	0.6

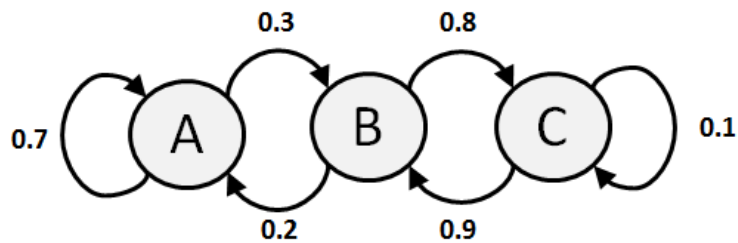


x2	P(X2)
0	0.8
1	0.2

x3	x1	x2	P(X3|X1,X2)
0	0	0	0.3
1	0	0	0.7
0	1	0	0.1
1	1	0	0.9
0	0	1	0.6
1	0	1	0.4
0	1	1	0.8
1	1	1	0.2

x4	x3	P(X4|X3)
0	0	0.2
1	0	0.8
0	1	0.5
1	1	0.5

Markov Chain Queries: Use the Markov chain transition diagram shown below to answer the following questions. Assume that $P(X_1 = A) = P(X_1 = B) = P(X_1 = C) = 1/3$.



1. $P(X_1 = A, X_2 = B, X_3 = C)$
2. $P(X_2 = C, X_3 = C | X_1 = B)$
3. $P(X_2 = C | X_1 = A)$
4. $P(X_3 = A | X_1 = A)$
5. Starting from $X_1 = B$, what are the most likely next three states?

Markov Chain Transition Diagrams: Given the following transition probability matrix, sketch the corresponding Markov chain transition diagram. Recall that p_{ij} (where i is the row index and j is the column index) indicates the probability of transitioning **from** state i **to** state j in the next time step.

$$p = \left[\begin{array}{c|ccc} & A & B & C \\ \hline A & 0.2 & 0.8 & 0 \\ B & 0 & 0.7 & 0.3 \\ C & 0.9 & 0 & 0.1 \end{array}\right]$$

Monty Hall problem with Bayesian Networks: The prize is behind any one of three closed doors (with equal probability). The player picks one of the three closed doors at random. The host opens one of the remaining two closed doors. The player can stick with her initial choice or switch doors.

Draw a Bayesian network to represent the Monty Hall problem (hint: use 3 random variables: location of prize, initial choice, door opened by host). Then, provide the factor tables for this Bayesian network.

To switch or not to switch: Compute the probability that the player wins if she switches and the probability she wins if she doesn't switch.