COMPSCI 514: ALGORITHMS FOR DATA SCIENCE

Andrew McGregor

Lecture 7

DISTINCT ELEMENTS RECAP

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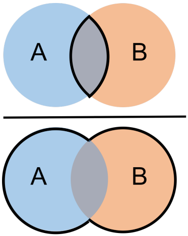
- **Median Trick:** If an algorithm returns a sufficiently accurate numerical answer with probability at least 3/4, run it $O(\log(1/\delta))$ times and take the median answer. This will have the required accuracy with probability at least $1 - \delta$.

Questions on distinct elements counting?

Jaccard Index: A similarity measure between two sets.

$$J(A, B) = \frac{|A \cap B|}{|A \cup B|} = \frac{\#\text{ shared elements}}{\#\text{ total elements}}.$$



Natural measure for similarity between bit strings – interpret an n bit string as a set, containing the elements corresponding the positions of its ones. $J(x, y) = \frac{\#\text{ shared ones}}{\text{total ones}}$.

SEARCH WITH JACCARD SIMILARITY

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- **All-pairs Similarity Search:** Have n different sets/bit strings and want to find all pairs with high Jaccard similarity. $\Omega(n^2)$ time if we check all pairs explicitly.

Will speed up via randomized locality sensitive hashing.

APPLICATIONS

Document Similarity:

- E.g., detecting plagiarism, copyright infringement, spam.
- Use Shingling + Jaccard similarity.

The quick brown fox jumped high. → { the quick brown, quick brown fox, brown fox jumped, fox jumped high }

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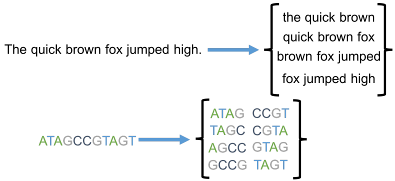
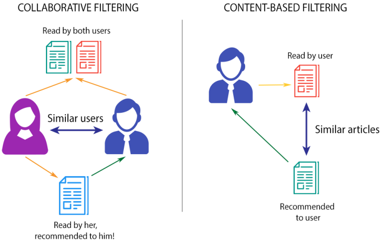


Online recommendation systems are often based on **collaborative filtering**.

Simplest approach: find similar users and make recommendations based on those users.



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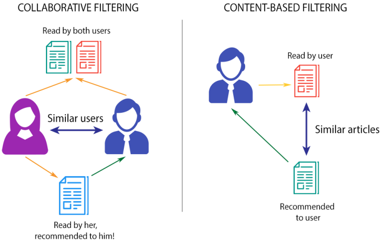

- Twitter: represent a user as the set of accounts they follow. Match similar users based on the Jaccard similarity of these sets. Recommend that you follow accounts followed by similar users. Netflix: look at sets of movies watched. Amazon: look at products purchased, etc.

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See Section 3.8.2 of *Mining Massive Datasets* for a discussion of a real world example involving 1 million customers. Naively this would be $\binom{1000000}{2} \approx 500$ billion pairs of customers to check!

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- **Fake Reviews**: Very common on websites like Amazon. Detection often looks for (near) duplicate reviews on similar products, which have been copied. 'Near duplicate' measured with shingles + Jaccard similarity.
- **Lateral phishing**: Phishing emails sent to addresses at a business coming from a legitimate email address at the same business that has been compromised.
  - One method of detection looks at the recipient list of an email and checks if it has small Jaccard similarity with any previous recipient lists. If not, the email is flagged as possible spam.

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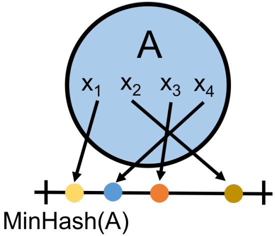

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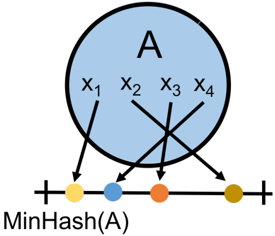

Identical to our distinct elements sketch!

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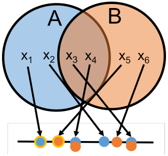

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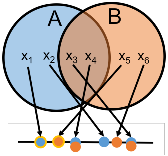

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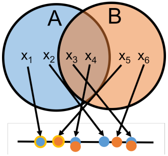

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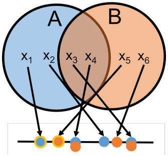

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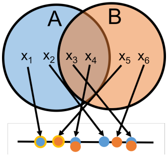

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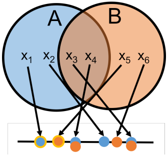

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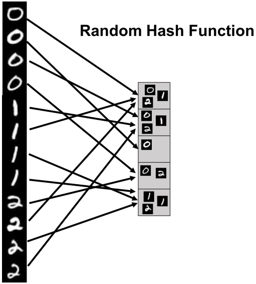

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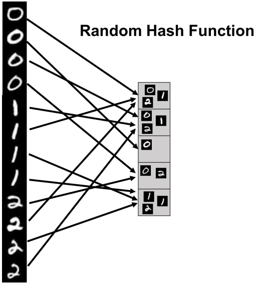

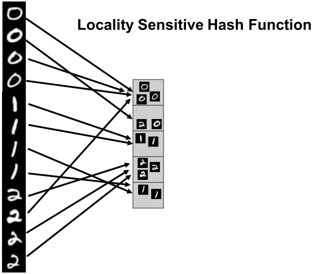

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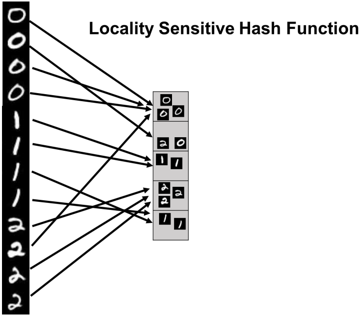

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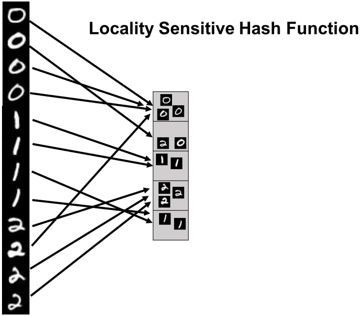

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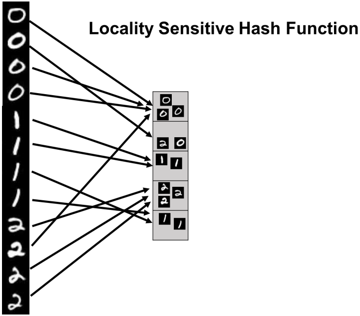

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Potential for a lot of false positives! Slows down search time.

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We want to balance a small probability of false negatives (a high hit rate) with a small probability of false positives (a small query time.)

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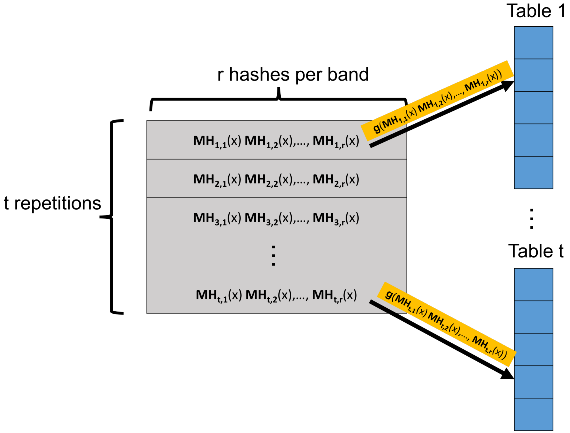

Create t hash tables. Each is indexed into not with a single MinHash value, but with r values, appended together. A length r signature.

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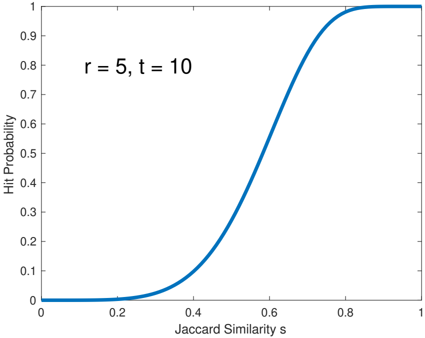

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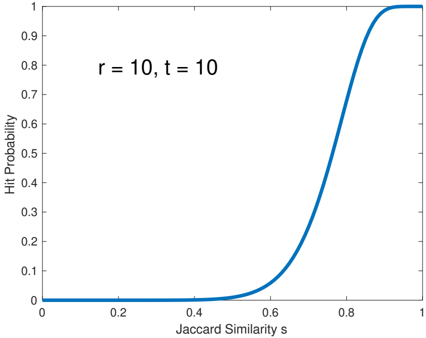

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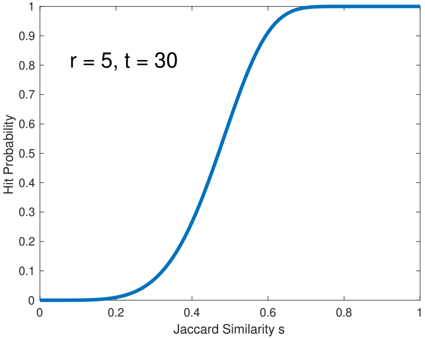

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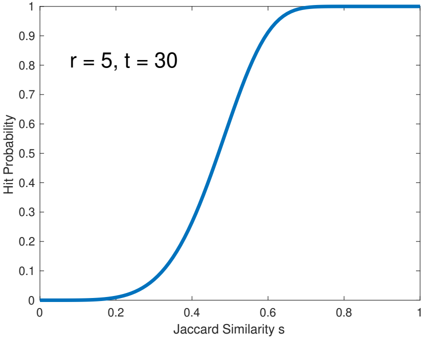

r and t are tuned depending on application. 'Threshold' when hit probability is $1/2$ is $\approx (1/t)^{1/r}$. E.g., $\approx (1/30)^{1/5} = .51$ in this case.

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