COMPSCI 514: ALGORITHMS FOR DATA SCIENCE

Andrew McGregor

Lecture 8

Jaccard Index: A similarity measure between two sets.

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Want Fast Implementations For:

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SEARCH WITH JACCARD SIMILARITY

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- **Near Neighbor Search:** Have a database of n sets and given a set A, want to find if it has high Jaccard similarity to anything in the database. $\Omega(n)$ time with a linear scan.
- **All-pairs Similarity Search:** Have n different sets and want to find all pairs with high Jaccard similarity. $\Omega(n^2)$ time if we check all pairs explicitly.

Will speed up via randomized locality sensitive hashing.

Goal: Speed up Jaccard similarity search.

MINHASHING

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Strategy: Use random hashing to map each set to a very compressed representation. Jaccard similarity can be estimated from these.

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- $\mathbf{s} := 1$
- For $x_1, \ldots, x_{|A|} \in A$
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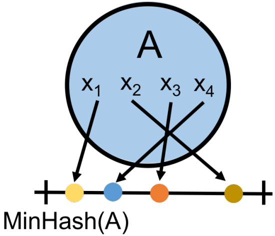

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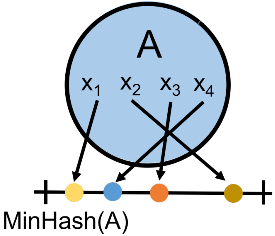

Identical to our distinct elements sketch!

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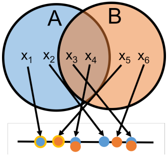

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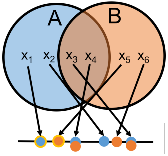

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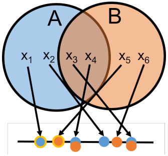

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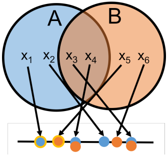

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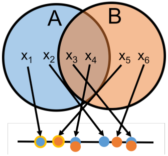

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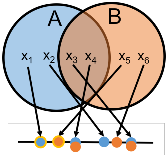

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\end{aligned}$$

Upshot: MinHash reduces estimating the Jaccard similarity to checking equality of a *single number*.

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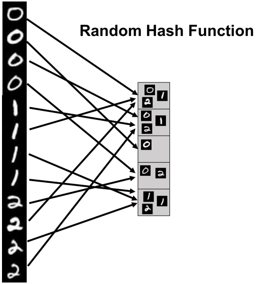

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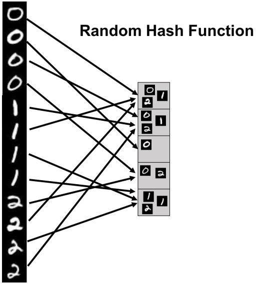
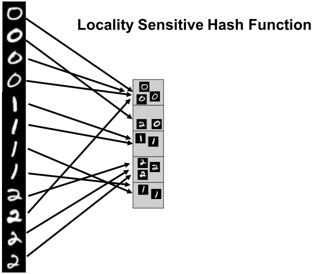
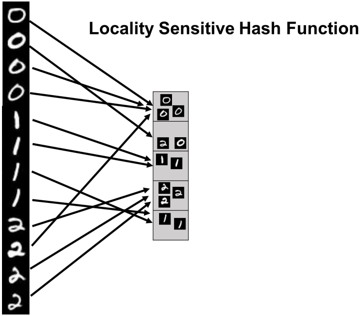




LSH FOR SIMILARITY SEARCH

How does locality sensitive hashing help for similarity search?



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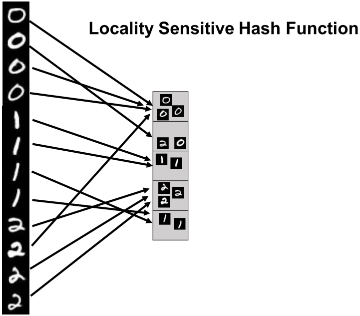

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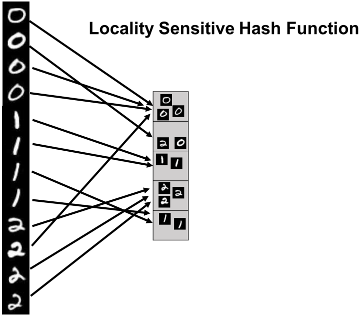

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- **All-pairs Similarity Search:** Scan through all buckets of the hash table and look for similar pairs within each bucket.

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Potential for a lot of false positives! Slows down search time.

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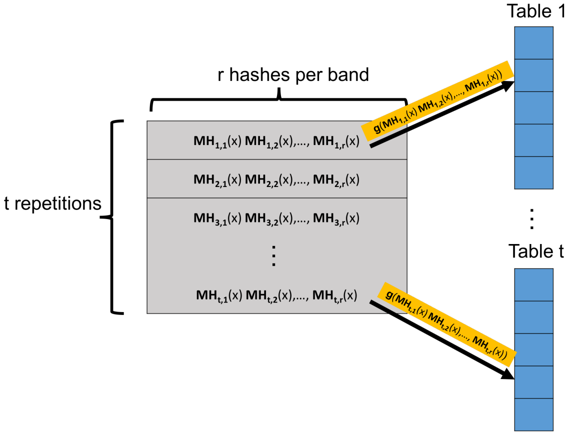

Create t hash tables. Each is indexed into not with a single MinHash value, but with r values, appended together. A length r signature.

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  $\Pr[MH_{i,1}(x), \ldots, MH_{i,r}(x) = MH_{i,1}(y), \ldots, MH_{i,r}(y)]$.

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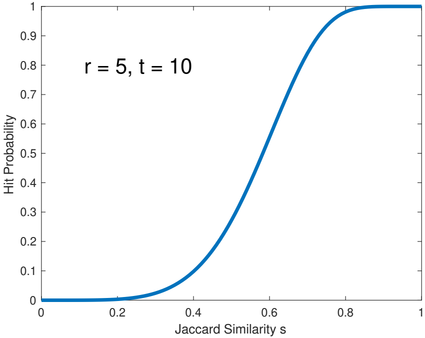

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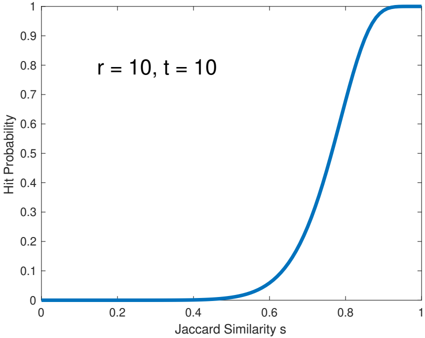

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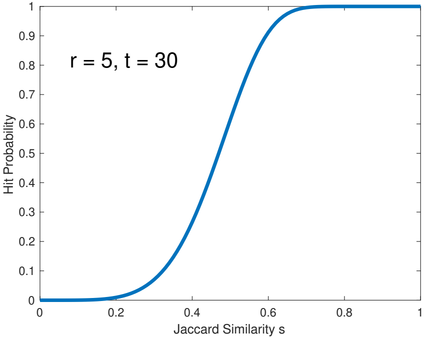

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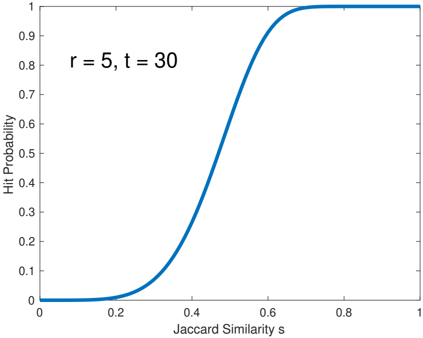

r and t are tuned depending on application. 'Threshold' when hit probability is $1/2$ is $\approx (1/t)^{1/r}$. E.g., $\approx (1/30)^{1/5} = .51$ in this case.

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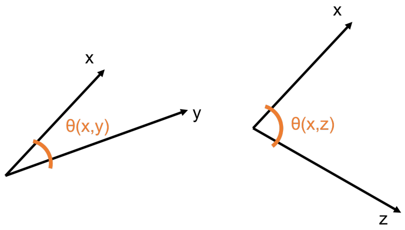

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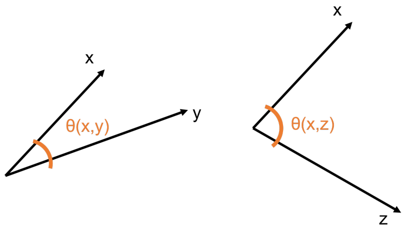

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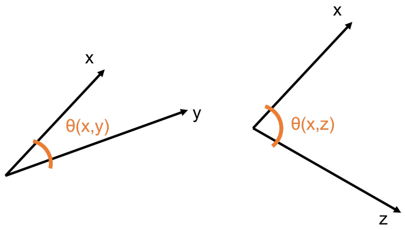

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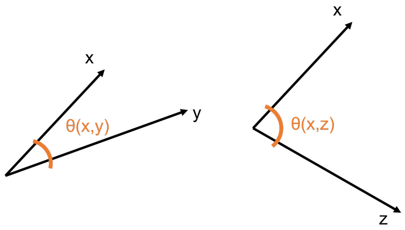

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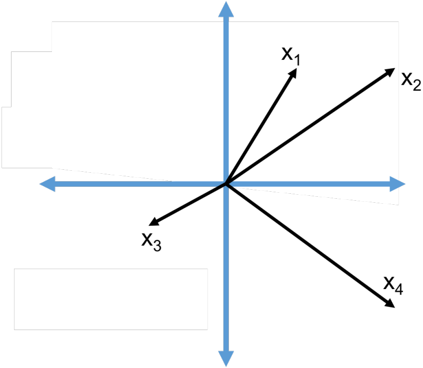

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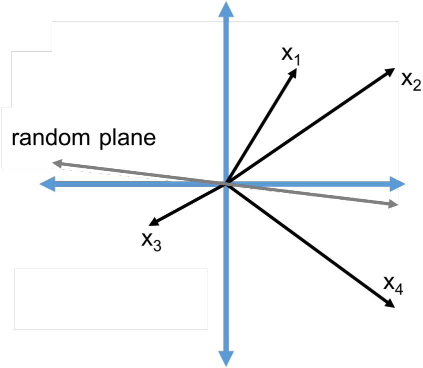

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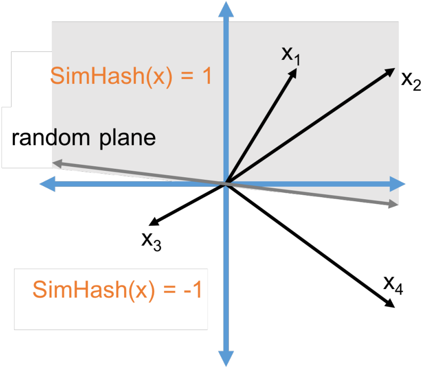

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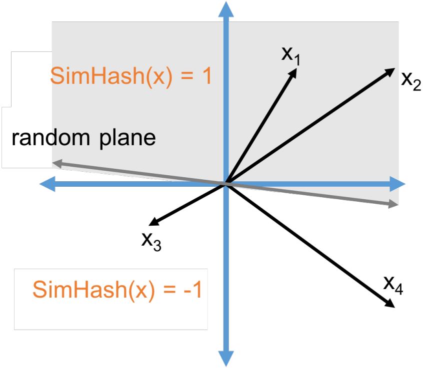

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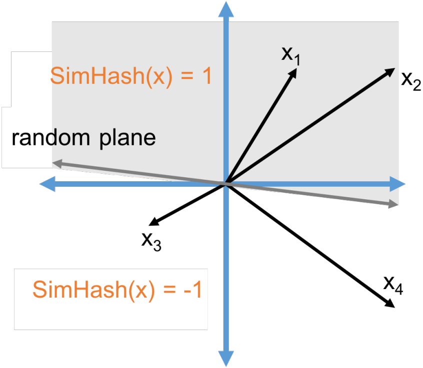

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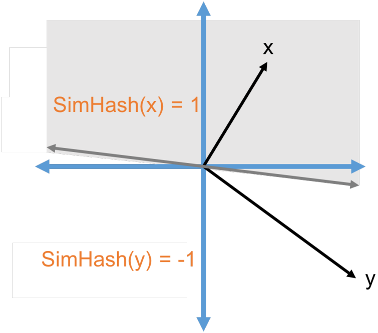

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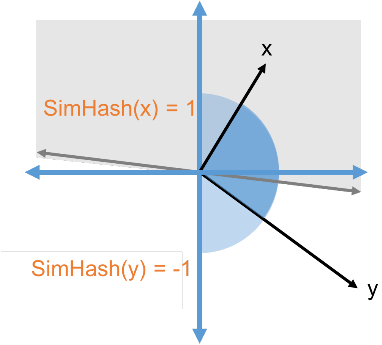

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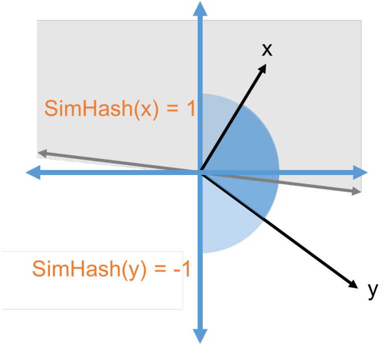

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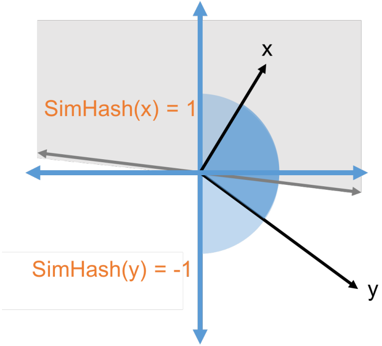

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- $\Pr[SimHash(x) = SimHash(y)] = 1 - \frac{\theta(x,y)}{180} \approx \cos\theta$ for small θ.

Questions on MinHash and Locality Sensitive Hashing?