CMPSCI 611: Advanced Algorithms
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Problem Set 1

Out: September 26, 2000
Due: October 3, 2000

Notes: *On homework assignments, you are allowed to discuss the questions with a small number of other people in the course. However, the emphasis of such discussions should be obtaining a solid understanding of the solutions to the assigned problem. Thus, you must destroy any notes from your discussions, and then write up the solutions on your own. For each problem, you must also list anyone you discussed that problem with (even briefly), and any other references you used.*

The homeworks are due at the beginning of class on the due date. Late submissions will be accepted only with special permission. Also, please take the time to write clear and concise answers. Credit will be reduced if answers are very unclear or long winded.

All questions count for the same amount of credit, although some are harder than others. Some of the questions may require quite a bit of thought, so I encourage you to start early.

1. a) Show how to multiply two complex integers (i.e., numbers of the form $a + bi$, where $i = \sqrt{-1}$, and a and b are integers) using only 3 integer multiplies.

   b) Strassen's algorithm is based on a way of multiplying two 2×2 matrices using seven multiplications. We might try to improve on this by developing a way to multiply two 4×4 matrices using less than the obvious 64 multiplications. How small would the number of multiplies have to be in order to do better than Strassen's algorithm?

   c) Here is how to multiply two 4×4 matrices $A = (a_{ij})$ and $B = (b_{ij})$ using only 48 multiplications. First compute $\alpha_1, \alpha_2, \alpha_3, \alpha_4, \beta_1, \beta_2, \beta_3, \beta_4$, where $\alpha_i = a_{i1}a_{i2} + a_{i3}a_{i4}$, and $\beta_i = b_{1i}b_{2i} + b_{3i}b_{4i}$. The elements of the product $C = (c_{ij})$ can then be computed as follows:

   $$c_{ij} = (a_{i1} + b_{2j})(a_{i2} + b_{1j}) + (a_{i3} + b_{4j})(a_{i4} + b_{3j}) - \alpha_i - \beta_j$$

   Verify that this method is correct, but also explain why it does not in fact lead to an improvement in runtime over Strassen's algorithm.

2. Problem 32-2 on page 797 of [CLR].

3. Problem 32-4 on page 798 of [CLR].

   Note: my copy of [CLR] has a typo in its description of this problem. In the end of the second sentence on page 799, "mod $p_{ij}(x)$" should be "mod $P_{ij}(x)$."

4. Matroids do not cover all cases where greedy algorithms can be used. For example, Section 17.1 of [CLR] describes the activity-selection problem, and proves that the algorithm "GREEDY-ACTIVITY-SELECTOR" provides the optimal solution for this problem.

   a) Define a subset system and a weight function such that the activity-selection problem corresponds to finding an independent set of maximum total weight.

   b) Show that the subset system described in part (a) is not a matroid.

   c) In lecture, we proved that if the greedy algorithm works for a subset system, then that subset system is a matroid. This might seem to contradict the combination of the proof that the greedy algorithm "GREEDY-ACTIVITY-SELECTOR" provides the optimal solution for the activity-selection problem and the result you have shown in part (b). Describe why there is no contradiction.