CMPSCI 611: Advanced Algorithms
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Problem Set 2

Out: October 10, 2001
Due: October 17, 2001

1. In lecture, we saw an algorithm for the bipartite matching problem. With some effort, the general technique we used can be extended to work for arbitrary graphs as well. In this question, you are asked to provide an algorithm for an easier problem: *directed* matchings in arbitrary graphs.

   Given a directed graph $G = (V, E)$, a directed matching is a subset of the edges such that the indegree of any node is at most 1, and the outdegree of any node is at most 1. Show how to use an efficient algorithm for the (undirected) bipartite matching problem to find the largest cardinality directed matching in an arbitrary graph.

2. Intersection of matroids.

   (a) Consider the following problem: we are given a graph $G = (V, E)$, a vertex $v \in V$, and an integer k, and we want to determine whether or not there is a spanning tree of G in which v has degree k or less. Show that this problem can be formulated as the intersection of two matroids. What does this imply about how efficiently the problem can be solved?

   (b) Consider now the following problem: we are given for each vertex i an integer k_i, and we want to determine whether there is a spanning tree of G in which each vertex i has degree at most k_i. Note that we are looking for a single tree that satisfies all of the degree requirements. Define two subset systems analogous to the two you used in part (a) that might be used to solve this problem. Show why one of these subset systems is no longer a matroid.

3. In lecture, we examined the "rooted tree" implementation of the Union-Find data structure. We saw that if we use both "union-by-size" and "path compression", then the total cost of any sequence of m Union and Find instructions, starting with n singleton sets, is $O(\alpha(n)(m + n))$.

   (a) Show that if we use *only* '"union-by-size", then the cost of the operations is at most $O(m \log n)$, and describe a sequence of operations that proves that this bound is asymptotically tight in the worst case.

   (b) A general analysis of the case where we use only "path compression" is quite difficult, so we shall consider only a special case here. Assume that we start with a number of singleton sets, and then we have a series of Union operations followed by m Find operations. Show that the total cost of the Find operations is $\Theta(m)$.

4. [CLR] Problem 16-3 (page 325) ***Edit distance.***

5. [CLR] Problem 16-4 (page 326) ***Planning a company party.***