CMPSCI 611: Advanced Algorithms
Micah Adler

Problem Set 1

Out: September 23, 2003
Due: September 30, 2003

Notes: *On homework assignments, you are allowed to discuss the questions with a small number of other people in the course. However, the emphasis of such discussions should be obtaining a solid understanding of the solutions to the assigned problem. Thus, you must destroy any notes from your discussions, and then write up the solutions on your own. For each problem, you must also list anyone you discussed that problem with (even briefly). You also must describe any other references you used.*

The homeworks are due at the beginning of class on the due date. Late submissions will be accepted only with special permission. Also, please take the time to write clear and concise answers. Credit will be reduced if answers are unclear or long winded.

All questions count for the same amount of credit, although some are harder than others. Some of the questions may require quite a bit of thought, so I encourage you to start early.

1. In this question, we shall obtain a more exact bound on the running time of matrix multiplication, and use this to determine at what value of n Strassen's algorithm starts to outperform the naïve matrix multiplication algorithm. In order to do so, we shall use the fact that the solution to the recurrence relation

$$\begin{aligned} T(n) &= aT\left(\frac{n}{b}\right) + cn^\alpha \quad (n > 1); \\ T(1) &= d \end{aligned}$$

is

$$T(n) = n^\beta \left(d + \frac{cb^\alpha}{a - b^\alpha}\right) - n^\alpha \left(\frac{cb^\alpha}{a - b^\alpha}\right),$$

where $\beta = \log_b a$.

a) Let $S(n)$ be the running time of Strassen's algorithm for multiplying two $n \times n$ matrices, where the entries in the matrix are integers, adding two integers requires 1 time unit, and multiplying two integers requires m time units.

Give an exact recurrence relation (i.e., without any Θ or O expressions) for $S(n)$. Use the formula above to derive an explicit expression for $S(n)$ in terms of m and n. You should assume that n is a power of 2.

b) Let $N(n)$ be the running time of the naïve matrix multiplication algorithm in the same model. Give an exact recurrence relation for $N(n)$, and derive an explicit expression for $N(n)$ in terms of m and n.

c) For the case where $m = 10$, determine n_0, the smallest power of 2 such that $S(n) \leq N(n)$ for all $n > n_0$. Do the same for the case where $m = 1$.

2. [CLRS] Exercises 33.4-1 through 33.4-4. Note that in 33.4-1 and 33.4-2, the array Y' is an array containing the points within δ of the line dividing the set of points into two halves. The points in Y' are sorted in order of their y-coordinate.

3. For this question, it is probably useful for you to first familiarize yourself with the Convolution Theorem (Page 790 of [CLRS], Theorem 32.8).

(a) The *positive wrapped convolution* of vectors $\mathbf{a}= (a_0, a_1, \ldots, a_{n-1})$ and $\mathbf{b}= (b_0, b_1, \ldots, b_{n-1})$ is the vector $\mathbf{c}= (c_0, c_1, \ldots, c_{n-1})$, where for $0 \leq i < n$,

$$c_i = \sum_{j=0}^{i} a_j b_{i-j} + \sum_{j=i+1}^{n-1} a_j b_{n+i-j}.$$

Show how to compute a positive wrapped convolution in time $O(n \log n)$.

(b) [CLRS], Exercise 30.2-8.

4. (a) [CLRS], Exercise 16.4-4. In your proof, you should demonstrate directly that the subset system satisfies the exchange property (and thus, you should not use the Cardinality theorem). What is the relationship between this matroid, and Example 4 given in class (the Directed Graph matroid)?

   (b) - (f) [CLRS], Problem 16-3, parts (b) - (f).