CMPSCI 611: Advanced Algorithms
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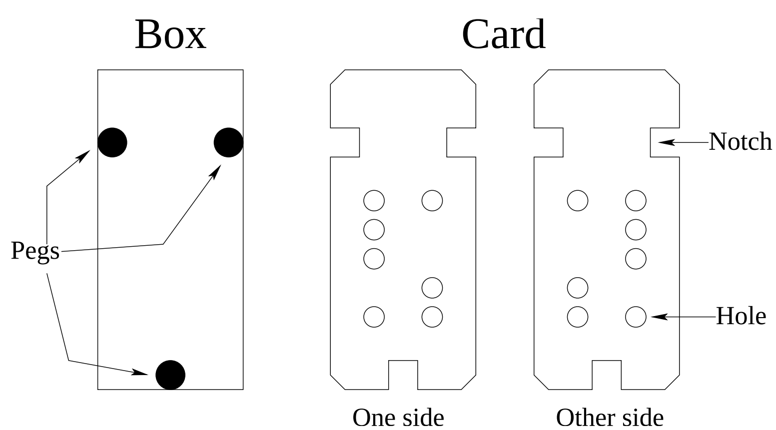
Problem Set 4

Out: November 18, 2003
Due: November 25, 2003

1. Prove that if $\mathbf{P} = \mathbf{NP}$,
   (a) then everything in **NP** is **NP**-Complete.
   (b) then we can design an algorithm A such that, given a set of integers $S = \{S_1, S_2, \ldots, S_n\}$ and a target integer t, A outputs "No Sum" if there is no subset of S that sums to t, and if there is such a subset, then A outputs such a subset. The algorithm A should run in time that is polynomial in n, regardless of the size of the integers in S.
   Note: keep in mind that **NP** is a class of decision problems, but the algorithm A must do more than just return a "Yes" or a "No".

2. Consider the following puzzle. You are given a box and a collection of cards as indicated in the figure below. The box has pegs in it, and the cards have notches. As a result, each card will fit in the box in either of two ways. Each card has two columns of holes, some of which may not be punched out. The puzzle is solved by placing all the cards in the box so as to completely cover the bottom of the box. This means that every hole position is blocked by at least one card that has no hole there. Conisider the $PUZZLE$ problem, where you are given a description of k cards, and are asked whether or not these cards form a puzzle which has a solution. Show that the $PUZZLE$ problem is **NP**-Complete.



3. Consider the decision version of the **3-SETS** problem:
   Input: Finite set $S = \{s_1, \ldots, s_n\}$, collection C of subsets of S such that each subset consists of exactly 3 elements of S, integer K.
   Question: Is there a subset $S' \subseteq S$ such that $|S'| \leq K$, and every subset in C contains at least one element in S'?

   Show that the **3-SETS** problem is NP-Complete. Hint: you can use without proof the fact that the **VERTEX-COVER** problem, defined as follows, is **NP**-Complete.
   Input: Graph $G = (V, E)$, integer K.
   Question: Is there a subset of the vertices $V' \subseteq V$ such that $|V'| \leq K$, and every edge in E is incident to at least one vertex in V'?

4. [CLRS] Problem 34-2 (page 1018).