CMPSCI 105 Fall 2009
Quiz #1 Solution Key
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1. Compute the complex division: **(16+11*i*) ÷ (3-2*i*)**

Answer: **2+5*i***

Discussion: To divide one complex number by another, you must multiply both numerator and denominator by a third complex number chosen so that the imaginary component of the denominator vanishes. This third number is the complex conjugate of the denominator, which is the same value except that the sign of the imaginary component is inverted. So, for this problem the complex conjugate of 3-2*i* is 3+2*i*. Multiplying both numerator and denominator by 3+2*i* does not change the result as it is equivalent to multiplying by 1. The problem is then:

$$\frac{(16+11i)(3+2i)}{(3-2i)(3+2i)}$$

The numerator is computed through the FOIL (first-outer-inner-last) method. This gives the four terms: $16\times3 + 16\times2i + 11i\times3 + 11i\times2i = 48 + 32i + 33i + 22i^2$, but because $i^2=-1$ this reduces to $26+65i$.

The denominator is computed the same way: $9 - 6i + 6i - 4i^2$, but the imaginary terms cancel, leaving $9 - 4i^2$, or 13.

The final result is $(26+65i)/13$, which reduces to:

2+5*i*

Scoring: 3 points. Accept only **2+5*i***. Remove 1 point for each error in the real component (the 2) or in the imaginary component (the +5*i*), up to the maximum of 3 points. Possible errors include wrong values, getting the sign wrong, not reducing i^2 to -1, not reducing $(26+65i)/13$, etc.

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2. On a computer with ***10-bit*** storage locations, what are the legal ranges of integer values for both ***signed*** and ***unsigned*** numbers?

Answer:

Signed:	$-2^9 \ldots +2^9-1$	=	-512…+511	(allow either answer)
Unsigned:	$0 \ldots 2^{10}-1$	=	0…1023	(allow either answer)

Discussion: For N bits the range for unsigned numbers is $0 \ldots 2^N-1$. When N=10, the range is $0 \ldots 2^{10}-1$ or 0…1024-1, which is 0…1023.

For N bits the range for signed numbers is $-2^{N-1} \ldots +2^{N-1}-1$. When N=10, the range is $-2^9 \ldots +2^9-1$ or -512…+512-1, which is -512…+511.

Scoring: 2 points. Assign 1 point for each answer. Remove ½ point for each minor error, up to the full point for each answer. These errors include specifying or computing the wrong exponent, forgetting the -1 in appropriate cases, off-by-1 errors, sign errors, etc. Allow equivalent exponential or numeric forms (e.g., allow either 2^{10} or 1024).

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3. Base conversions:

A. Convert the base 5 number $\mathbf{132_5}$ to base 10.

B. Convert the base 10 number $\mathbf{157_{10}}$ to base 2.

C. Convert the base 2 number $\mathbf{1111010010010_2}$ to base 16 using the bit-partitioning method.

Answers:

A. 42_{10}

B. 10011101_2

C. $1E92_{16}$

Discussion:

A. $132_5 = 1\times5^2 + 3\times5^1 + 2\times5^0 = 1\times25 + 3\times5 + 2\times1 = 25+15+2 = 42_{10}$.

B. Using the division method:

157 ÷ 2 = 78 R 1 (the remainder is the right-most bit of the answer)
78 ÷ 2 = 39 R 0
39 ÷ 2 = 19 R 1
19 ÷ 2 = 9 R 1
9 ÷ 2 = 4 R 1
4 ÷ 2 = 2 R 0
2 ÷ 2 = 1 R 0
1 ÷ 2 = 0 R 1 (the remainder is the left-most bit of the answer)

C. Use the bit-partitioning method to split the binary number 1111010010010 into 4-bit packets 0001-1110-1001-0010 (leading zeroes are necessary on the leftmost packet). Convert each 4-bit packet separately:

$0001 = 0\times2^3 + 0\times2^2 + 0\times2^1 + 1\times2^0 = 0\times8 + 0\times4 + 0\times2 + 1\times1 = 0+0+0+1=1.$

$1110 = 1\times2^3 + 1\times2^2 + 1\times2^1 + 0\times2^0 = 1\times8 + 1\times4 + 1\times2 + 0\times1 = 8+4+2+0=14=E.$

$1001 = 1\times2^3 + 0\times2^2 + 0\times2^1 + 1\times2^0 = 1\times8 + 0\times4 + 0\times2 + 1\times1 = 8+0+0+1 = 9.$

$0010 = 0\times2^3 + 0\times2^2 + 1\times2^1 + 0\times2^0 = 0\times8 + 0\times4 + 1\times2 + 0\times1 = 0+0+2+0 = 2.$

Scoring: 3 points. Assign 1 point per answer. Remove ½ point per answer for off-by-1 errors. Remove ½ point per answer if the students supply the correct solution framework, but get the wrong numeric answer.

4. Add together the following two binary numbers and show the binary result (do NOT convert them to base 10):

```
10001110011010001101
11000111001000111010
```

Answer: `10101010110001100011`1

Discussion: Line up the numbers and add them from right to left, but remember that 1+1 is 0 with a carry, and 1+1+1 is 1 with a carry:

```
  10001110011010001101
+ 11000111001000111010
 101010101100011000111
```

Scoring: 2 points. Remove 1 point for cases where the student copied the numbers down incorrectly but got the correct sum for those numbers. Remove ½ point for each bit in error, up to two total points.
