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## CMPSCI 240 Reasoning Under Uncertainty Discussion 3

1. You are a contestant on a game show and are given two questions to answer. You must choose which one to answer first. The probability you will get Question 1 right is 0.8 . The probability you will get Question 2 right is 0.5 . Answering Question 1 right wins you $\$ 100$, while answering Question 2 wins you $\$ 200$. No matter which question you answer first, you must get that question right in order to attempt to answer the other question. Which question should you answer first in order to maximize the expected value of your winnings?

Let random variable $X_{1}$ denote the amount of money you win if you choose Question 1 first and let $X_{2}$ denote the amount of money you win if you choose Question 2 first.
(a) Draw two sequential descriptions (tree diagrams)—one for answering Question 1 first and one for answering Question 2 first. Use these diagrams to find $\mathbb{E}\left[X_{1}\right]$ and $\mathbb{E}\left[X_{2}\right]$.
(b) Which question should you answer first and why?
(c) Now we will generalize the problem. Let $p_{1}$ and $p_{2}$ be the probabilities of correctly answering Questions 1 and 2, respectively, and let $v_{1}$ and $v_{2}$ be the amounts of the corresponding prizes. Under what conditions should you answer Question 1 first?
2. You are offered the chance to play the following game. A fair coin will be tossed until the first tail appears. If $n$ tosses are needed (including the tail), you will win $2^{n}$ dollars.
(a) What is the expected value of playing this game?
(b) How much would you be willing to pay to play this game?

